



Path Signature : 一种新的序列特征表示方法及其在图文识别中的应用

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2018.12.20

Springtime for AI: The rise of Deep Learning

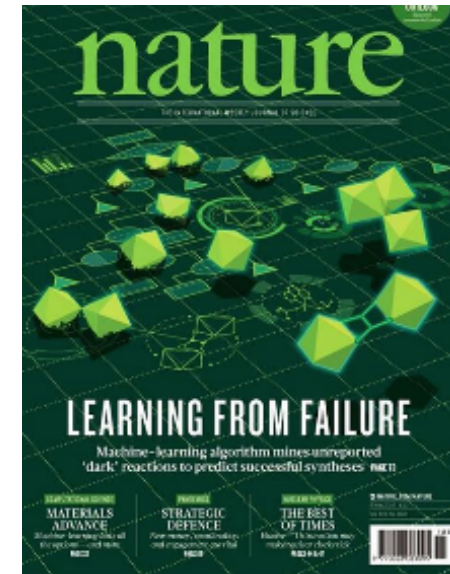
2015.7.17



2016.1.28



2016.5.5



2017.2.2



2017.7.7



2018.12.7

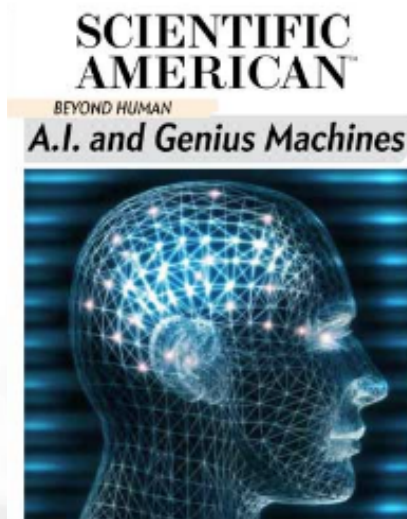


Springtime for AI

WHY DEEP LEARNING IS SUDDENLY CHANGING YOUR LIFE

“FORTUNE”: The Deep Learning Revolution

2016.10.01 Front page article of USA “Fortune” magazine



(June 1, 2016)

当然，也有一些不同的观点...



- “DL差不多就是民工活。调来调去，刷来刷去....缺乏清澈干净的内在美感”
- **“性能不够、加层来凑”**
- **Many hyperparameters to tune - e.g. number of layers, number of hidden units, optimization algorithm, activation function, etc.**
- **Blackbox - difficult to understand what the DNNs are “thinking”**
- **They can be fooled with antagonistic examples**
- ...
- “其实，既不用赞美也不用批评深度学习”

Deep Learning is everywhere?



**Deep Learning is
everywhere. CNNs are
everywhere!**
(or soon will be)

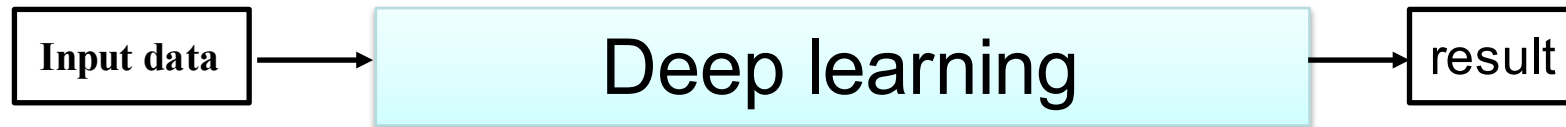


Why deep learning is so successful?

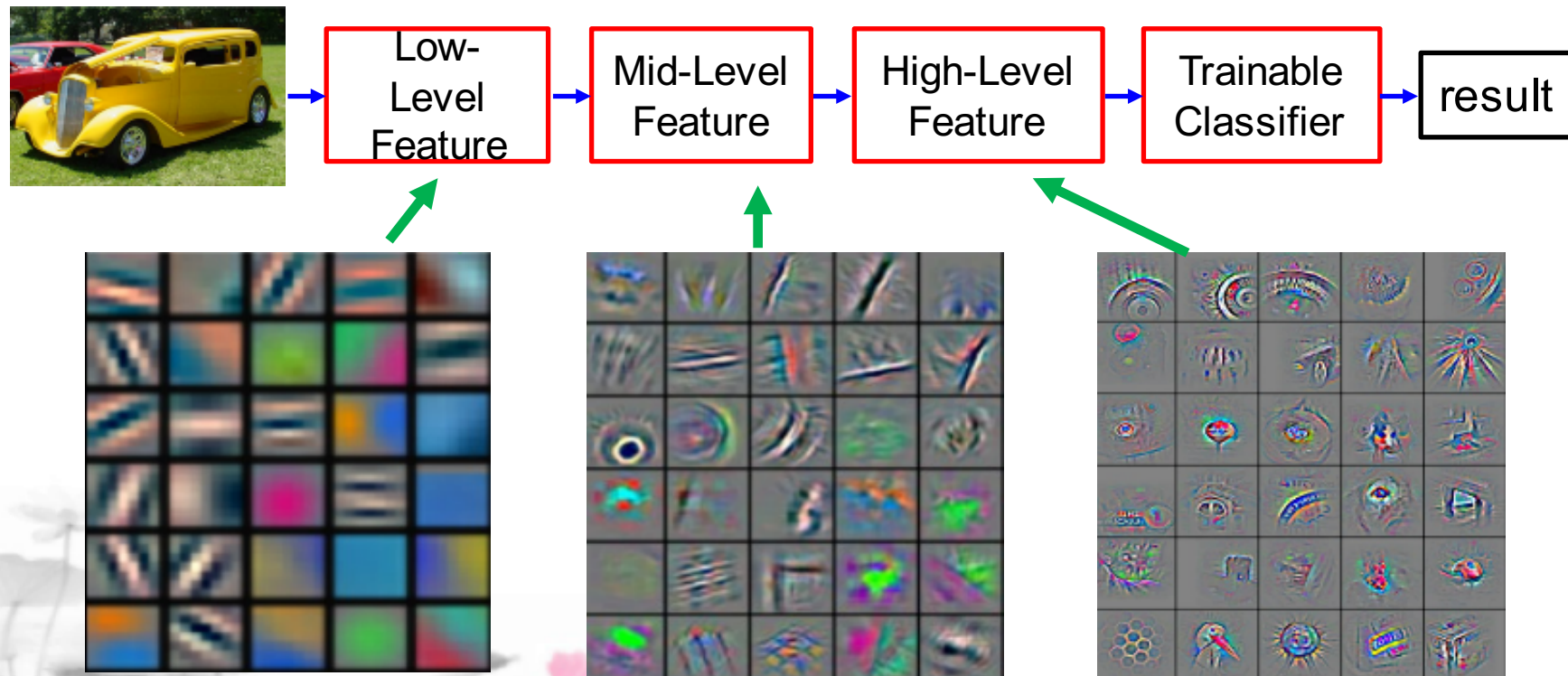


Deep Learning vs Trad. Machine Learning

■ Deep learning approach

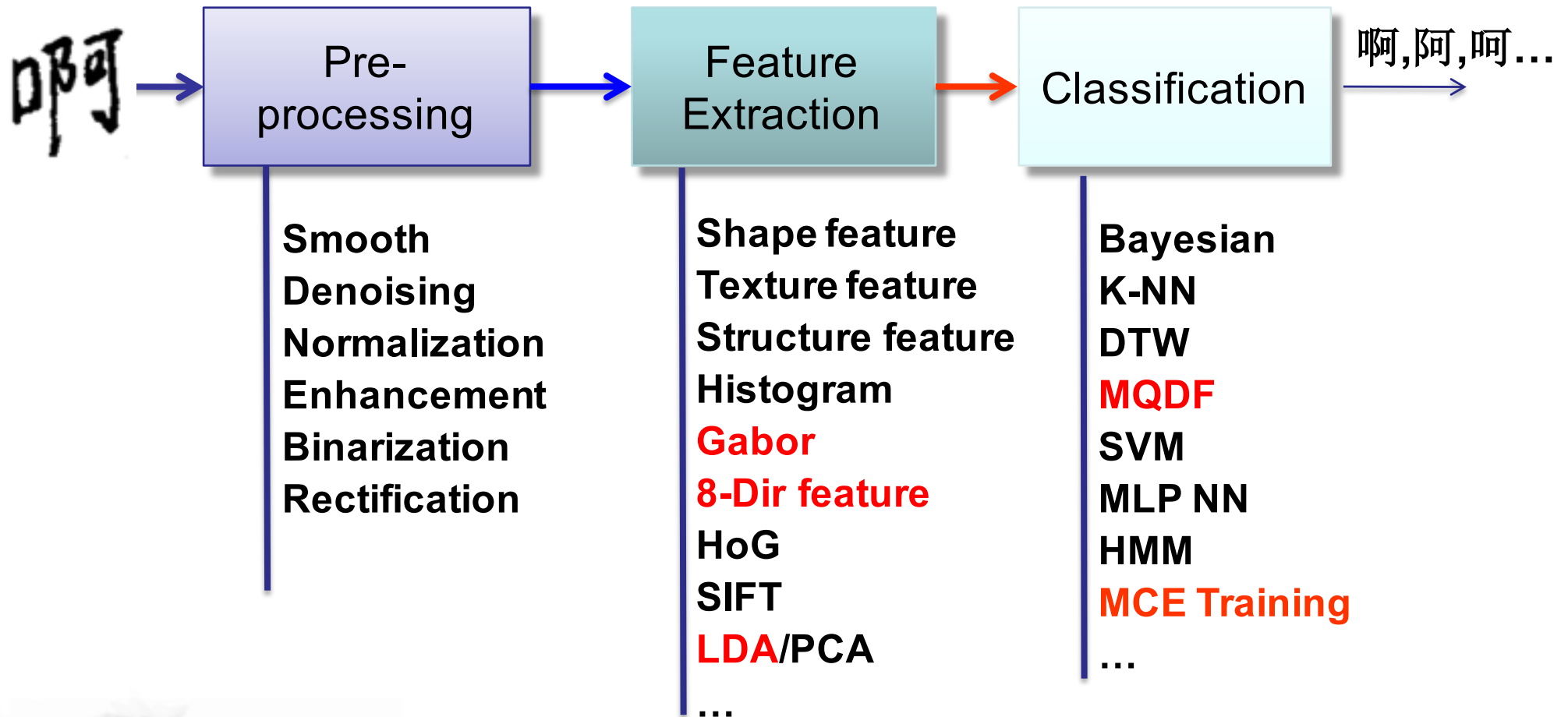


■ Traditional machine learning approach



Feature visualization from [Zeiler & Fergus 2013]

Literature Review : Traditional Methods



- **Typical methods:**

- Kimura et al, IEEE TPAMI 1987; Liu et al, IEEE TNN 2004;
- Liu et al, IEEE TPAMI 2004; Bai & Huo ICDAR 2005;
- Long & Jin, PR 2008; Liu et al, PR 2013; Du & Huo, PR 2014

Literature Review : Deep Learning Approach

- **Booming of Deep Learning since 2012**
 - New technologies: Dropout, ReLU, batch normalization, GoogleNet, ResNet, ...
 - Fast development of GPU hardware
 - The available of Big Data
 - The available of many open source tools such as Caffe, Tensorflow, PyTorch...

啊



Deep Learning
(CNN, DNN, RNN, LSTM, BLSTM, ...)

啊, 阿, 呵...



- **Typical methods:**
 - D. Ciresan, U. Meier, J. Schmidhuber, CVPR 2012; B. Graham, TR 2013
 - WX Yang LW Jin et al, PR 2016;
 - SX Lai, LW Jin, PRL 2017
 - XY Zhang, Y. Bengio, CL Liu, PR 2017
 - XY Zhang, F Yin et al, IEEE TPAMI 2018
 - Z Xie, Z Sun, LW Jin, IEEE TPAMI 2018

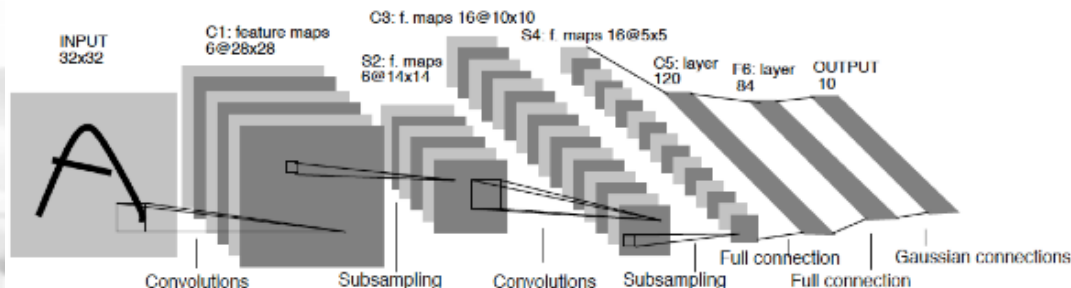
A long history of applications of CNN/RNN for character recognition

- Y. LeCun, B. Boser, et.al, NIPS 1989.
- Y. Bengio, Y. Lecun, Neural Computation 1995.
- S. Hochreiter, J. Schmidhuber, LSTM, *Neural Computation* 1997
- Y. LeCun, L. Bottou, Y. Bengio et al. *Proceedings of the IEEE*, 1998 (LeNet5)
- PY Simard, et al, ICDAR 2003 (CNN method, MNIST error rate: 0.4%)
- A Graves, J. Schmidhuber, NIPS 2008
- A Graves, M Liwicki, et al, IEEE TPAMI 2009 (LSTM+CTC)
- Ciresan D, Meier U, Schmidhuber J, CVPR 2012
- B. Graham, arXiv 2013 (ICDAR 联机手写汉字识别竞赛第一名)
- ...

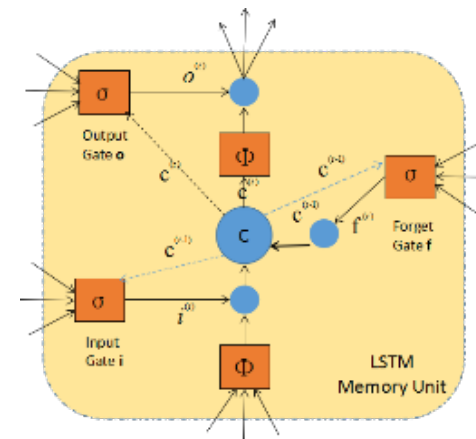
PROC. OF THE IEEE, NOVEMBER 1998

Gradient-Based Learning Applied to Document Recognition

Yann LeCun, Léon Bottou, Yoshua Bengio, and Patrick Haffner

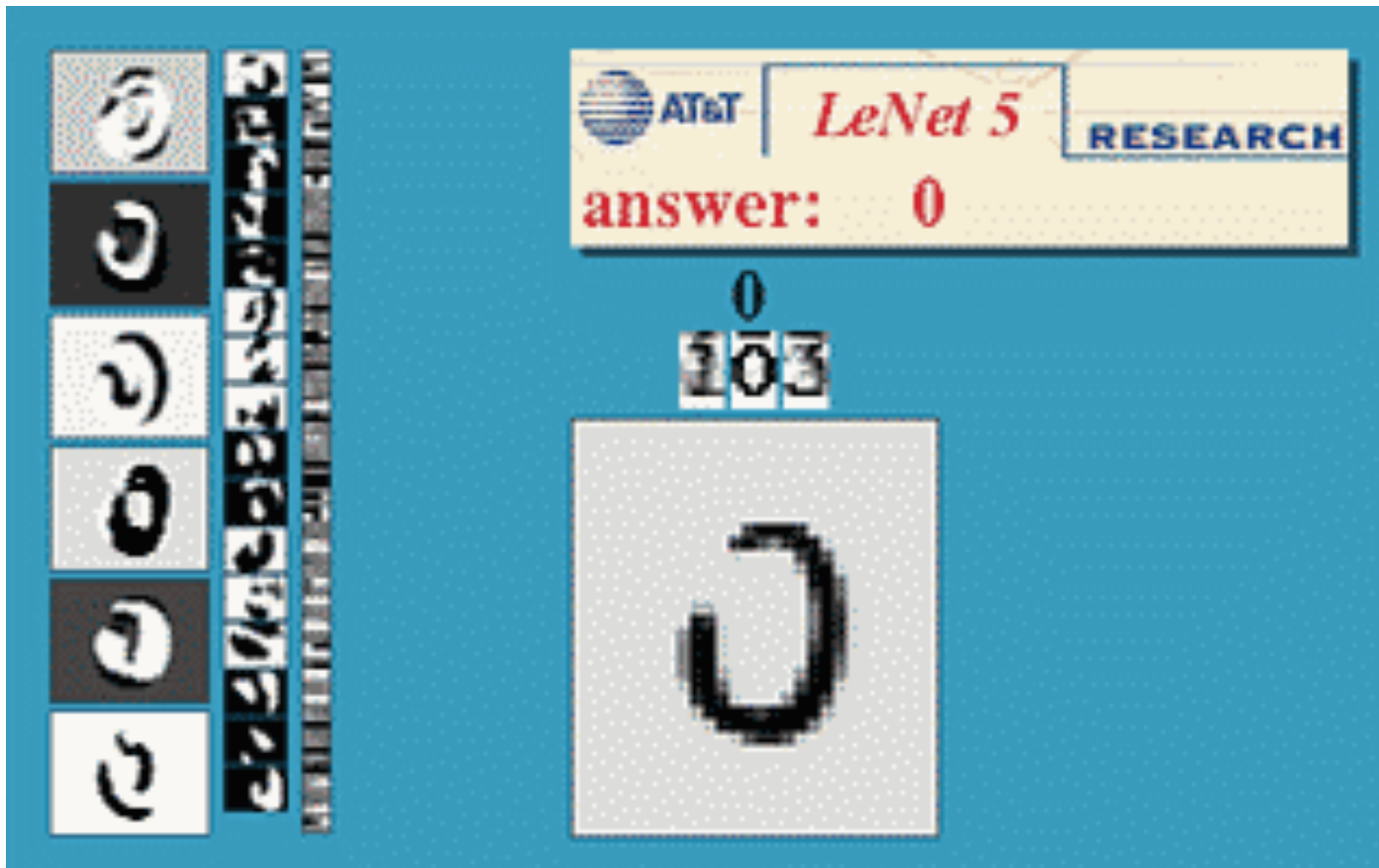


☆ 99 Cited by 14621 Related articles All 55 versions



A long history of applications of CNN/RNN for handwriting recognition

LeNet-5 CNN for handwriting recognition



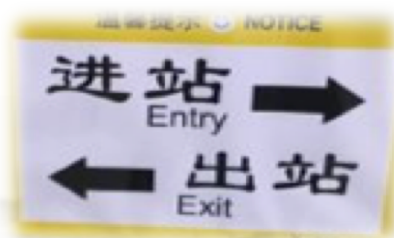
<http://yann.lecun.com/exdb/lenet/index.html>

Y. LeCun, L. Bottou, Y. Bengio, and P. Haffner. *Proceedings of the IEEE*, 1998

文字是信息交流及感知世界最重要的手段



秋天该很好
你若尚在场



Can we understand the meaning of this image without any text description?



文字的重要性

给一张图，
如果上面有文字，
绝大多数情况下，
图中的文字最有信息量！



图不重要

看👉字



Handwritten Chinese Character Recognition

❑ Challenges of online handwritten Chinese character/text recognition (**OHCCR/OHCTR**)

● Large-scale vocabulary

- 6763 classes in the GB2312-80 standard
- 27533 classes in the GB18010-2000 standard

● Diversity of handwriting styles

- More than 1.3 billions people use Chinese

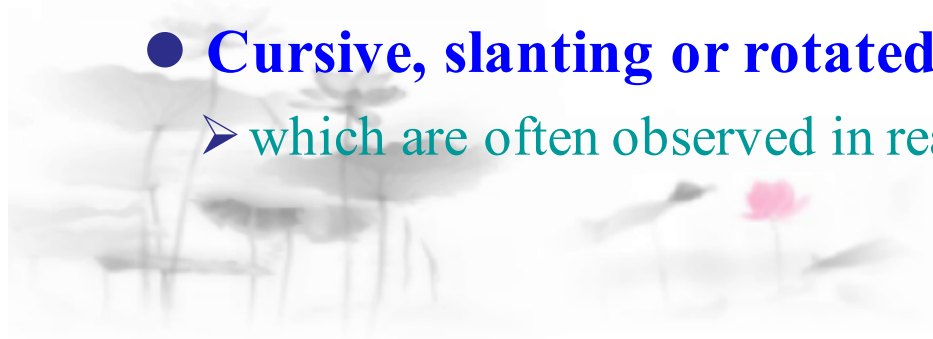
● Many similar and confusable Chinese characters, e.g.:

- “已-己”, “口-囗(wéi)”, “汨-汨(mì)-汨(gǔ)”, “毀-毀”, “沫-沫(mèi)” “睢-睢(huī)”
- “尢-尢”, “睛-晴”, “海-悔”, “绞-纹”, “莱-菜”

● Ambiguous segmentation in text line

● Cursive, slanting or rotated writing styles

- which are often observed in real world handwritten samples



characters (cont.)

[illegible]

开考15分钟以后, 缺考考生由监考老师粘贴条形码, 并用2B铅笔填涂右边的缺考标记

单项选择题（共27分，每小题3分）

1 [A] [B] ☒ [D]

2 [A] [B] [C]

3 [B] [C] [D]

4 ~~REDACTED~~ [B] [C] [D]

5 [A] [B] [C] [D]

6 [A] [B] [C]

13 [A] [B] ☒ [C] [D]

14 [A] [B] [C] ☒ [D]

15 [A] [B] ☒ [C] [D]

非选择题

二、(共36分)

(一) 7. (10分) 译文 (1) 我之所以常令手下经过西阙,是想告诉友人你并不在特殊时期不登楼。

(2) 以前赠给你的财物,你接受了,是不想拒绝我,如今一年了尚未开启,你的贞节可见啊。

(二) 8. (5分) 1. “争”是“竞”的意思，作者运用拟人手法，形象地从侧面写出船行驶之快，山河争相映入眼帘。2. “乱”是“集乱”的意思，雨来乱无章地冲刷着楼台，雨中来乱，舟雨之集乱，船之摇晃，烘托出作者内心感，颠天万，感情混乱。

9. (6分) 1. “人语朱离远”写诗人新到一地遇到新人，表现诗人寄居他乡的惆怅；2. “棹歌”表现诗人复职的喜悦，3. “归心猿”却欲愁”表现诗人对突然复职的适应，以及对复职后前途迷茫的担忧。所以这两句表现了诗人内心的惆怅、喜悦及迷茫。

请在各题目答题区域内作答，超出黑色矩形边框限定区域的答题无效

高三语文答题卡 第1页 (共4页)

[illegible]

手写汉字识别竞赛&Deep Learning

- ❑ ICDAR 2011 HCCR competition (Beijing)
 - Online HCCR first place: VO team with NN: 95.77%
 - Online HCCR (fourth place): IDSIA team with CNN: 93.06%
 - Offline HCCR first place: IDSIA team with CNN: 92.12%
- ❑ The MCDNN model (CVPR 2012)
 - Online HCCR: 95.39%
 - Offline HCCR: 93.50%
- ❑ ICDAR 2013 HCCR competition (USA)
 - Online HCCR winner:
 - Sparse CNN (DeepCNet) (by Dr. Graham, Warwick University): 97.39%
 - Offline HCCR winner :
 - Fujitsu Team: 94.77%



Sparse CNN for HCCR (ICDAR 2013 winner)

Sparse arrays of signatures for online character recognition

Benjamin Graham

University of Warwick

b.graham@warwick.ac.uk

November 12, 2013

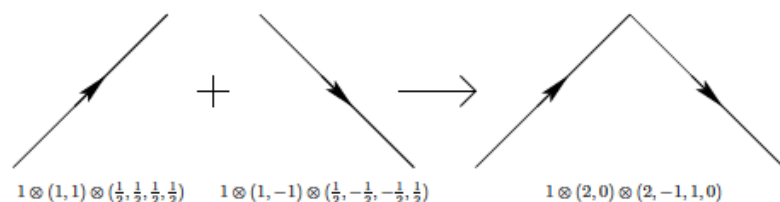


Figure 2: Concatenating paths and the corresponding $m = 2$ signatures.

3 The sparse signature grid representation

The expression of the information contained in a path in the form of iterated integrals was pioneered by K.T. Chen [3]. More recently, path signatures have been used to solve differential equations driven by rough paths [11, 12]. The signature extracts enough information from the path to solve any linear differential equation and uniquely characterizes paths of finite length [5].

The signature has been used in sound compression [10]. A stereo audio recording can be seen as a highly oscillating path in $\mathbb{R}^2$. Storing a truncated version of the path signature allows a version of the audio signal to be reconstructed.

Although computing the signature of a path is easy, the inverse problem is rather more difficult. The limiting factor in [10] was the lack of an efficient algorithm for reconstructing a path from its truncated signature when $m > 2$. We side-step the inverse problem by learning to recognize the signatures directly.

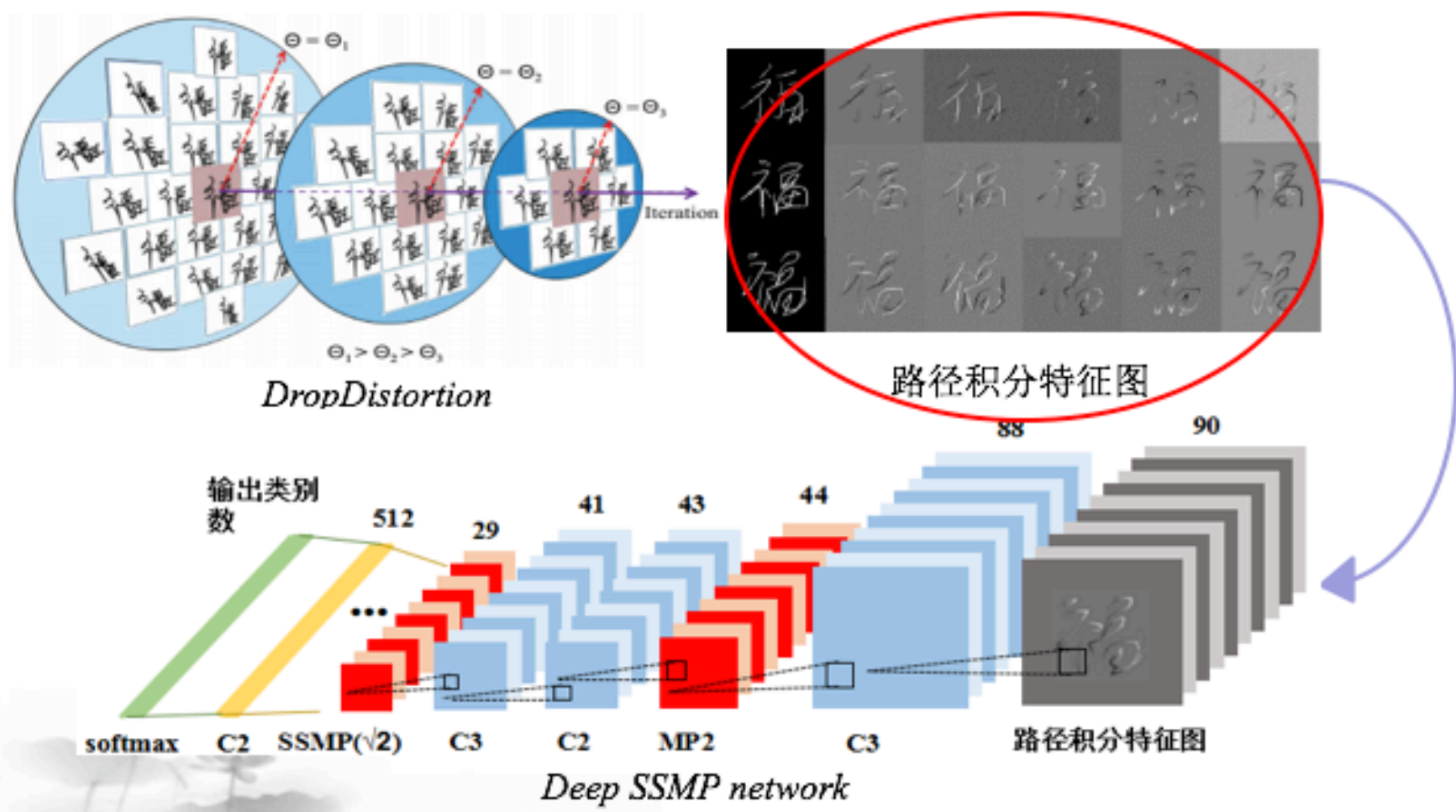
3.1 Computation of the path signature

Let $[S, T] \subset \mathbb{R}$ denote a time interval and let $V = \mathbb{R}^d$ with $d \geq 2$ denote the

I joined Facebook AI Research (FAIR) as a research scientist in July 2016.

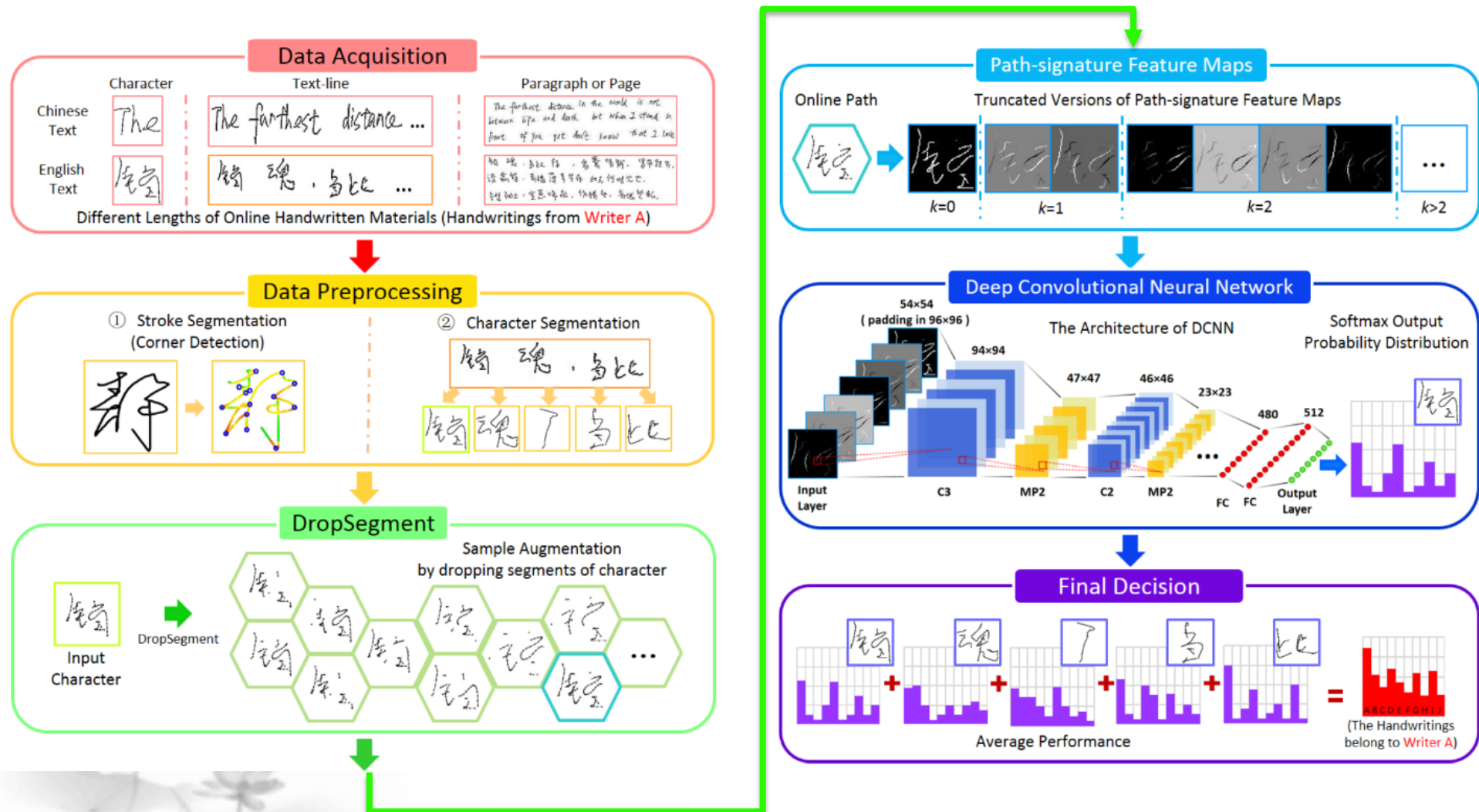
Prior to Facebook, I studied mathematics, and in particular probability theory relating to spatial phenomenon, as an undergraduate at Oxford, PhD student (Cambridge, supervised by Professor Geoffrey Grimmett) and post doc (UBC Vancouver and ENS Paris). Whilst working in the Department of Statistics at the University of Warwick, I became interested in machine learning and took part in a few Kaggle competitions.

High Performance Large Scale OHCCR with CNN and Path Signature



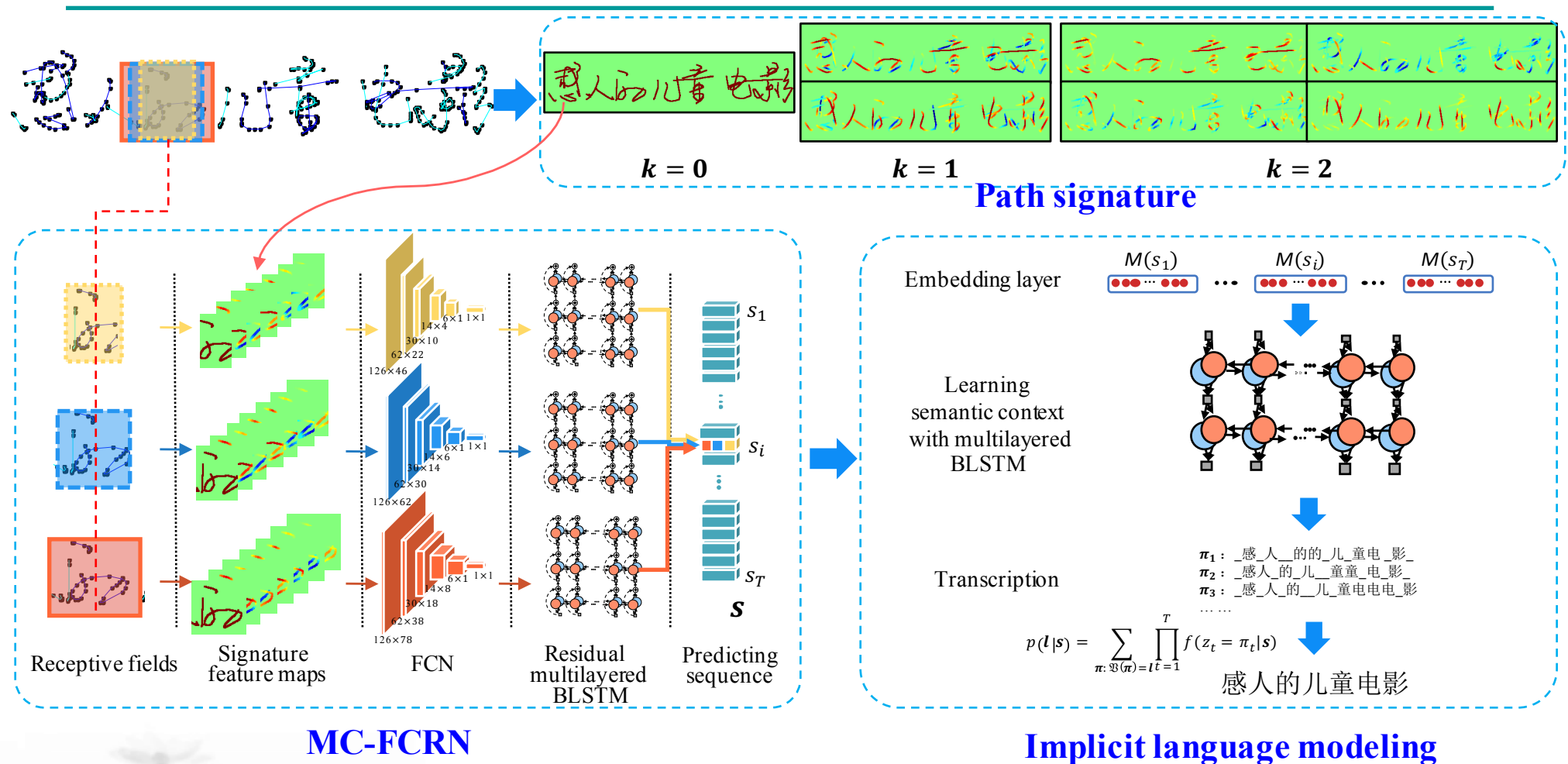
- WX Yang, LW Jin, et al, **ICDAR 2015**
- WX Yang, LW Jin, et al, **Pattern Recognition 2016**
- SX Lai, LW Jin, WX Yang, **Pattern Recognition Letters, 2017**

DeepWriterID: system diagram



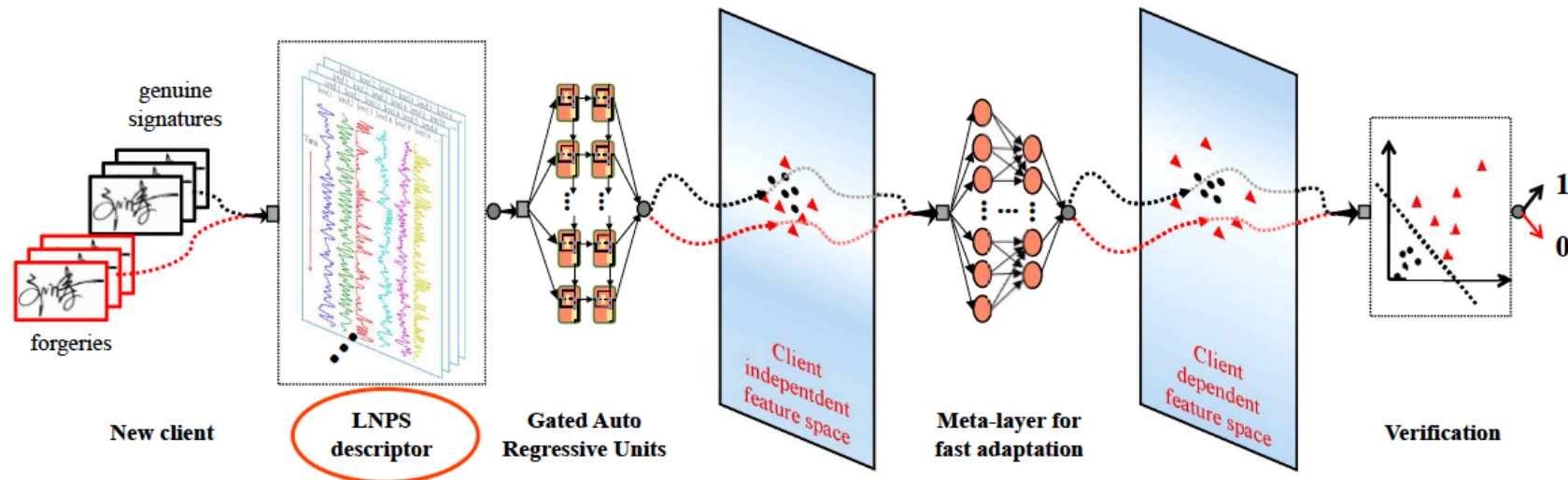
W.X. Yang, L.W. Jin, et.al., IEEE Intelligent Systems 2016

OHCTR with Fully Convolutional Recurrent Network



ZC Xie, ZH Shun, LW Jin, H Ni, T Lyons, Learning Spatial-Semantic Context with Fully Convolutional Recurrent Network for Online Handwritten Chinese Text Recognition, **IEEE TPAMI**, 2018.

Recurrent Adaptation Networks for Online Signature Verification



$$I^k(X) = \int_{0 < t_1 < \dots < t_k < T} 1 dX_{t_1} \otimes \dots \otimes dX_{t_k},$$

$$S(X)|_m = [1 \quad I^1(X) \quad I^2(X) \quad \dots \quad I^m(X)]^T$$

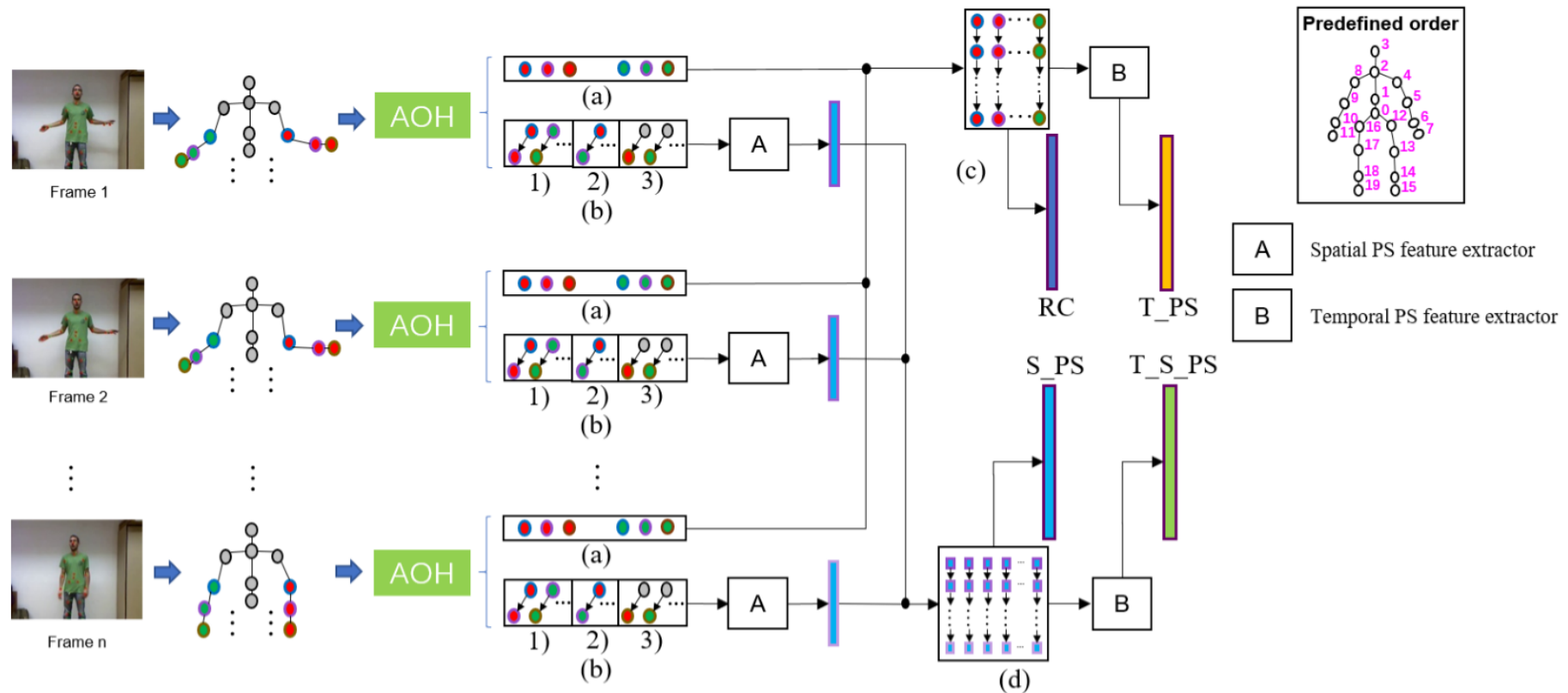


Length-normalized Path Signature (LNPS):

$$S(X)|_m^{LN} = \left[1 \quad \frac{I^1(X)}{L(X)} \quad \frac{I^2(X)}{L^2(X)} \quad \dots \quad \frac{I^m(X)}{L^m(X)} \right]^T$$

Songxuan Lai, Lianwen Jin, Recurrent Adaptation Networks for Online Signature Verification, **IEEE Transactions on Information Forensics and Security (TIFS)**, 2019.

Skeleton-based hand gesture recognition



C. Li, X Zhang, L. Liao, L. Jin, W Yang, "Skeleton-based Gesture Recognition Using Several Fully Connected Layers with Path Signature Features and Temporal Transformer Module", **AAAI 2019**

An Brief Introduction to Path Signature



Path Signature

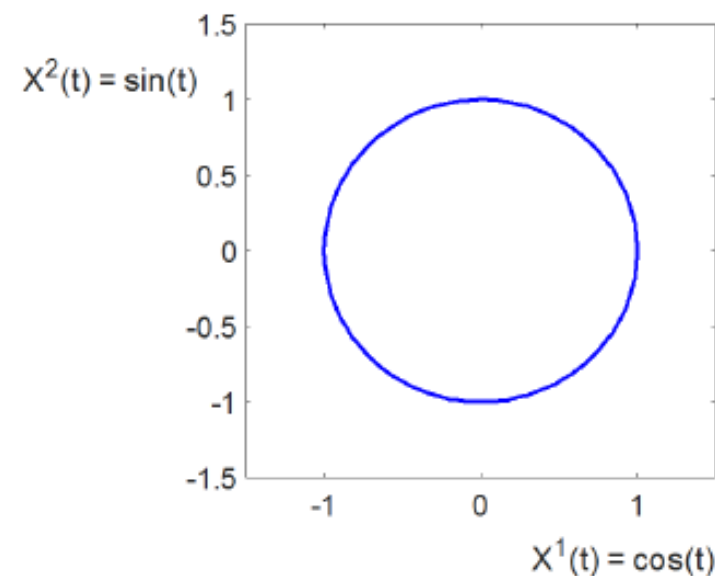
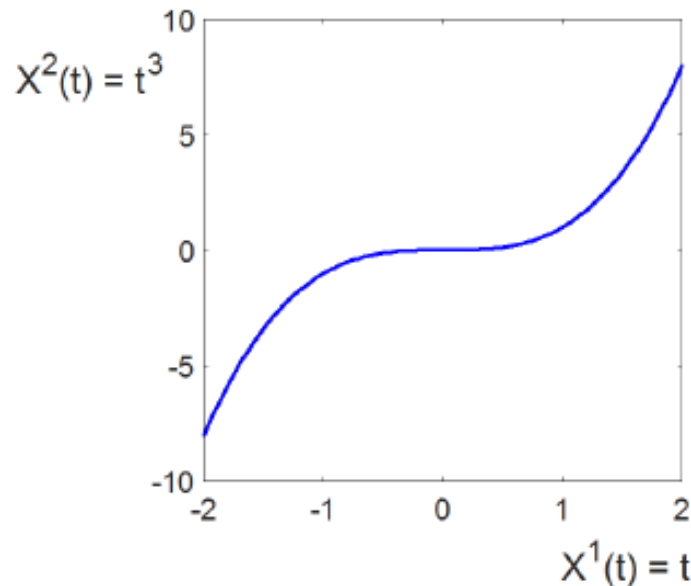
- The path signature theory was mainly developed by Terry Lyons and his team in recent years [3-6]
- The signature path feature was first introduced by Ben Graham to the field of handwritten characters. [1]
 - A novel feature representation for HCCR
 - Extract features contained in a path by iterated integrals
- Path signature involves iterated integrals of finite length **path** (e.g., **online handwriting trajectories**) in order to extract valuable information in that finite path.

1. B. Graham, arXiv 2013 (**winner of the ICDAR 2013 HCCR competition**)
2. K.T. Chen, Trans. Amer. Math. Soc. 1958
3. T. Lyons, Z. Qian, System control and rough paths, 2002.
4. T. Lyons, arXiv 2014.
5. D. Levin, T. Lyons, H. Ni, arXiv 2013
6. B. Hambly, T. Lyons, **Ann. of Math., 2010**

Paths in Euclidean space

A path X in $\mathbb{R}^d$ is a continuous mapping from some interval $[a, b]$ to $\mathbb{R}^d$, written as $X : [a, b] \rightarrow \mathbb{R}^d$.

- Example of two-dimensional smooth paths



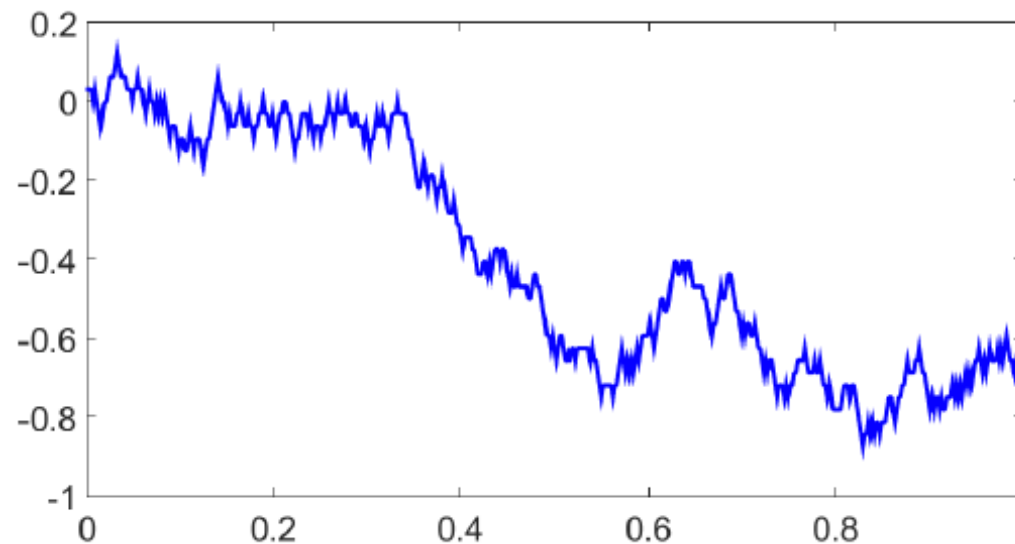
left panel: $X_t = \{X_t^1, X_t^2\} = \{t, t^3\}$, $t \in [-2, 2]$

right panel: $X_t = \{X_t^1, X_t^2\} = \{\cos t, \sin t\}$, $t \in [0, 2\pi]$

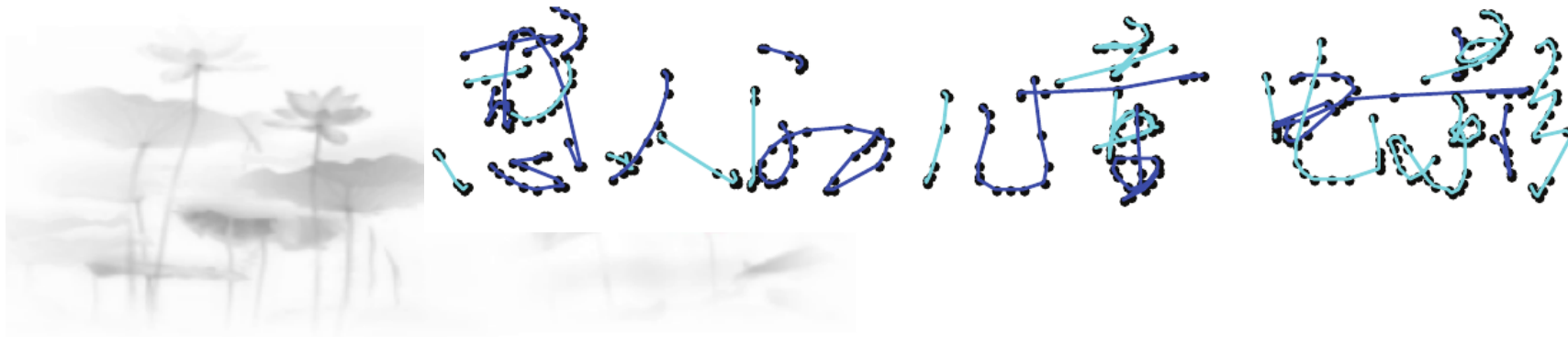
Paths in Euclidean space

- Example 2: Stock prices

$$X_t = \{X_t^1, X_t^2\} = \{t, f(t)\}, t \in [t_s, t_e]$$



- Example 3: Online handwriting trajectories



Path integral

For one-dimensional paths $X : [a,b] \rightarrow \mathbb{R}$ and $Y : [a,b] \rightarrow \mathbb{R}$, one can integrate path Y against path X :

$$\int_a^b Y_t dX_t = \int_a^b Y_t \dot{X}_t dt.$$

- Example 1: Consider $Y_t=1$ for all $t \in [a,b]$. The path integral of Y against any path $X : [a,b] \rightarrow \mathbb{R}$ is simply the increment of X

$$\int_a^b dX_t = \int_a^b \dot{X}_t dt = X_b - X_a.$$

- Example 2: A two-dimensional path Example.

$$X_t = \{X_t^1, X_t^2\} = \{t^2, t^3\}, t \in [0,1].$$

Path integral of X_t^1 against X_t^2 :

$$\int_0^1 X_t^1 dX_t^2 = \int_0^1 t^2 3t^2 dt = \frac{3}{5}.$$



Path Signature

For a path $X : [a,b] \rightarrow \mathbb{R}^d$ ($X_t = \{X_t^1, \dots, X_t^d\}$) where each $X^i : [a,b] \rightarrow \mathbb{R}$ is a real-valued path.

- The 1-fold iterated integral:

$$S(X)_{a,b}^i = \int_{a < t < b} dX_t^i = X_b^i - X_a^i, \quad i \in \{1, \dots, d\}.$$

- The 2-fold iterated integral:

$$\begin{aligned} S(X)_{a,b}^{i,j} &= \int_{a < t < b} S(X)_{a,t}^i dX_t^j \\ &= \int_{a < t_1 < t_2 < b} dX_{t_1}^i dX_{t_2}^j, \quad i, j \in \{1, \dots, d\}. \end{aligned}$$

Note for $t \in [a,b]$, $S(X)_{a,t}^i$ is itself a real-valued path, therefore we can integrate it against path X^j (recall the path integral).



Path Signature (PS)

- The k-fold iterated integral:

$$\begin{aligned} S(X)_{a,b}^{i_1, \dots, i_k} &= \int_{a < t < b} S(X)_{a,t}^{i_1, \dots, i_{k-1}} dX_t^{i_k} \\ &= \int_{a < t_1 < \dots < t_k < b} dX_{t_1}^{i_1} \dots dX_{t_k}^{i_k}, \quad i_1, \dots, i_k \in \{1, \dots, d\}. \end{aligned}$$

Definition:

The **signature of a path** $X: [a,b] \rightarrow \mathbb{R}^d$ is the collection (**infinite series**) of all the iterated integrals of X , e.g.:

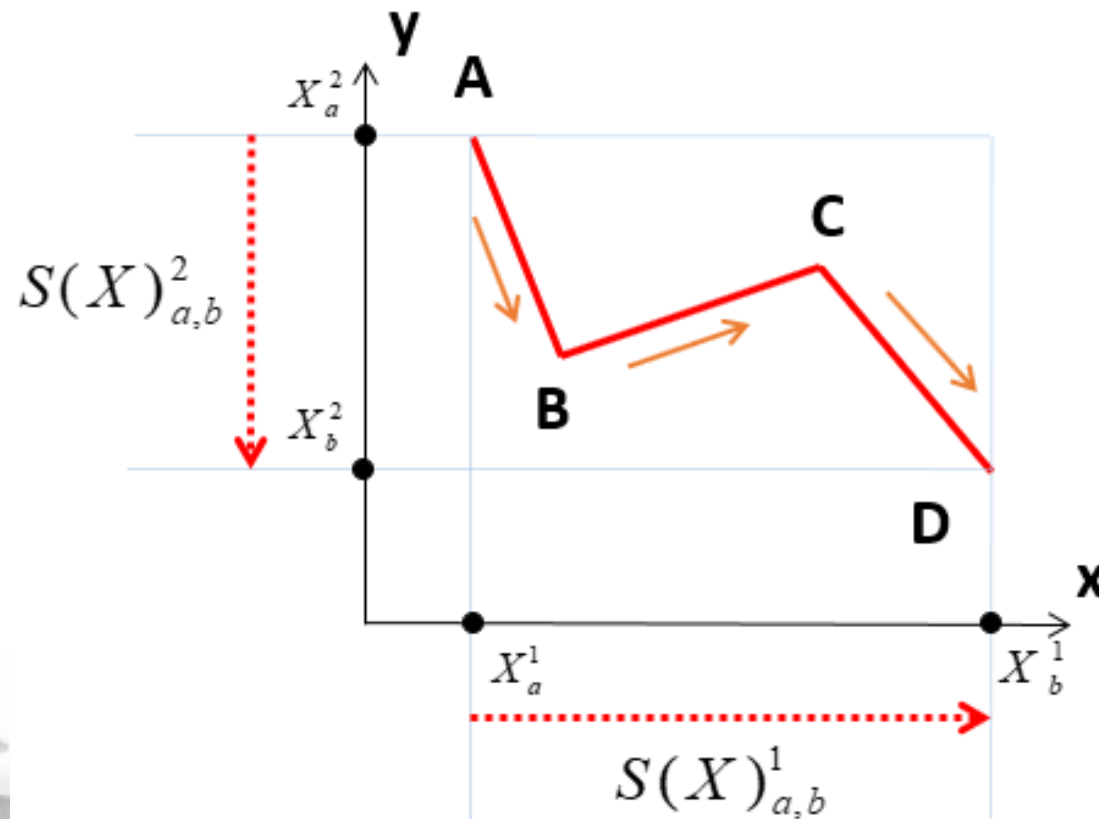
$$S(X)_{a,b} = (1, S(X)_{a,b}^1, \dots, S(X)_{a,b}^d, S(X)_{a,b}^{1,1}, S(X)_{a,b}^{1,2}, \dots)$$

where the zero-th term, by convention, is equal to 1.

Path signature in 2D

- The 1-fold (1-level) terms (Increments)

$$S(X)_{a,b} = (1, \boxed{S(X)_{a,b}^1, S(X)_{a,b}^2}, S(X)_{a,b}^{1,1}, S(X)_{a,b}^{1,2}, S(X)_{a,b}^{2,1}, S(X)_{a,b}^{2,2}, \dots)$$



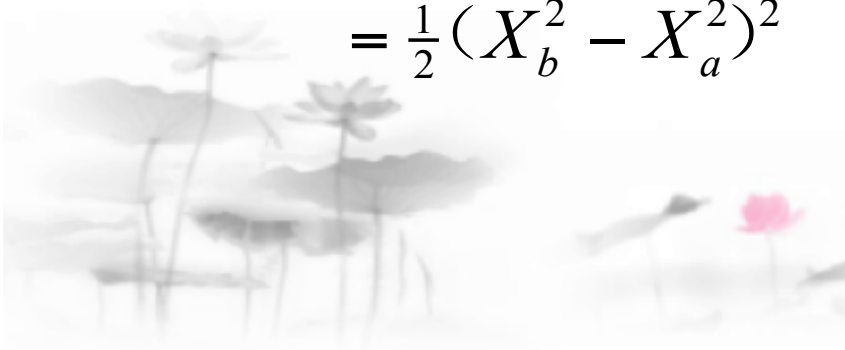
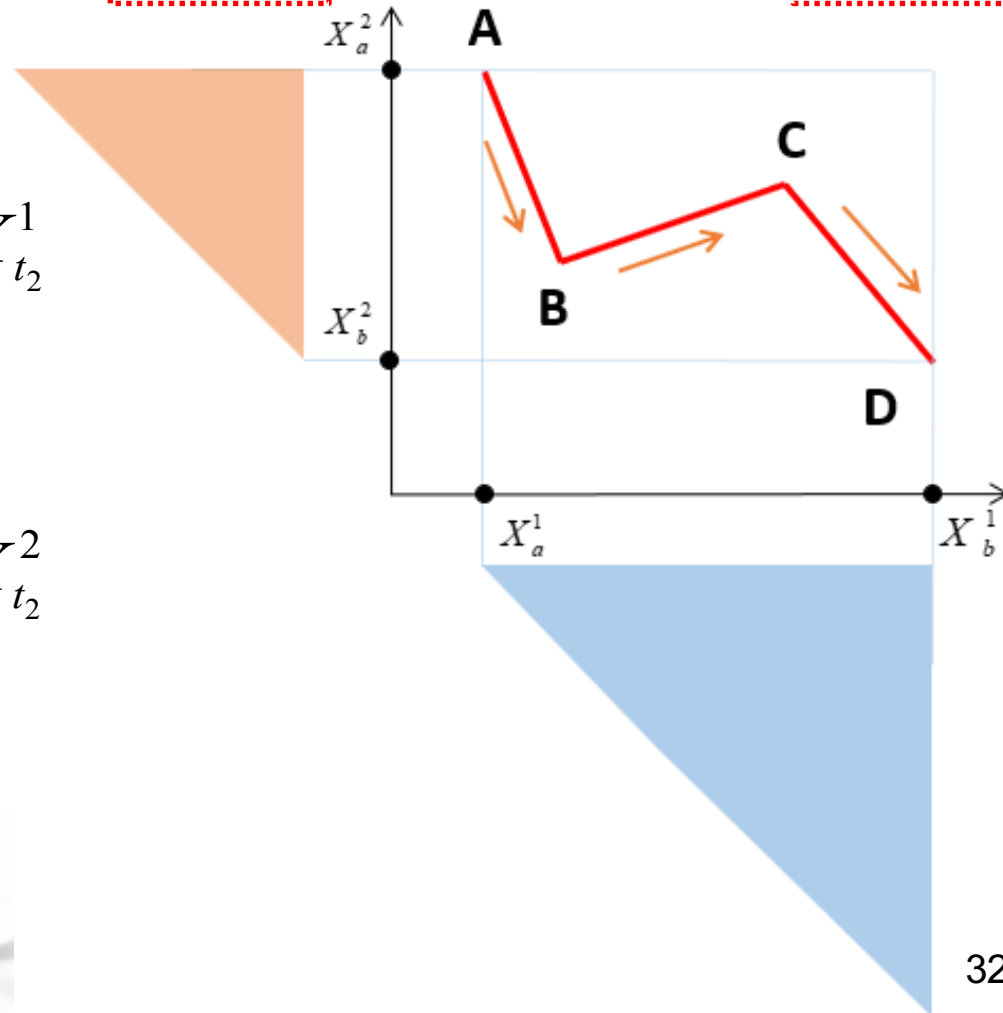
Path signature in 2D

- The 2-fold (level) terms (Areas)

$$S(X)_{a,b} = (1, S(X)_{a,b}^1, S(X)_{a,b}^2, \boxed{S(X)_{a,b}^{1,1}}, S(X)_{a,b}^{1,2}, S(X)_{a,b}^{2,1}, \boxed{S(X)_{a,b}^{2,2}}, \dots)$$

$$\begin{aligned} S(X)_{a,b}^{1,1} &= \int_{a < t_1 < t_2 < b} dX_{t_1}^1 dX_{t_2}^1 \\ &= \frac{1}{2} (X_b^1 - X_a^1)^2 \end{aligned}$$

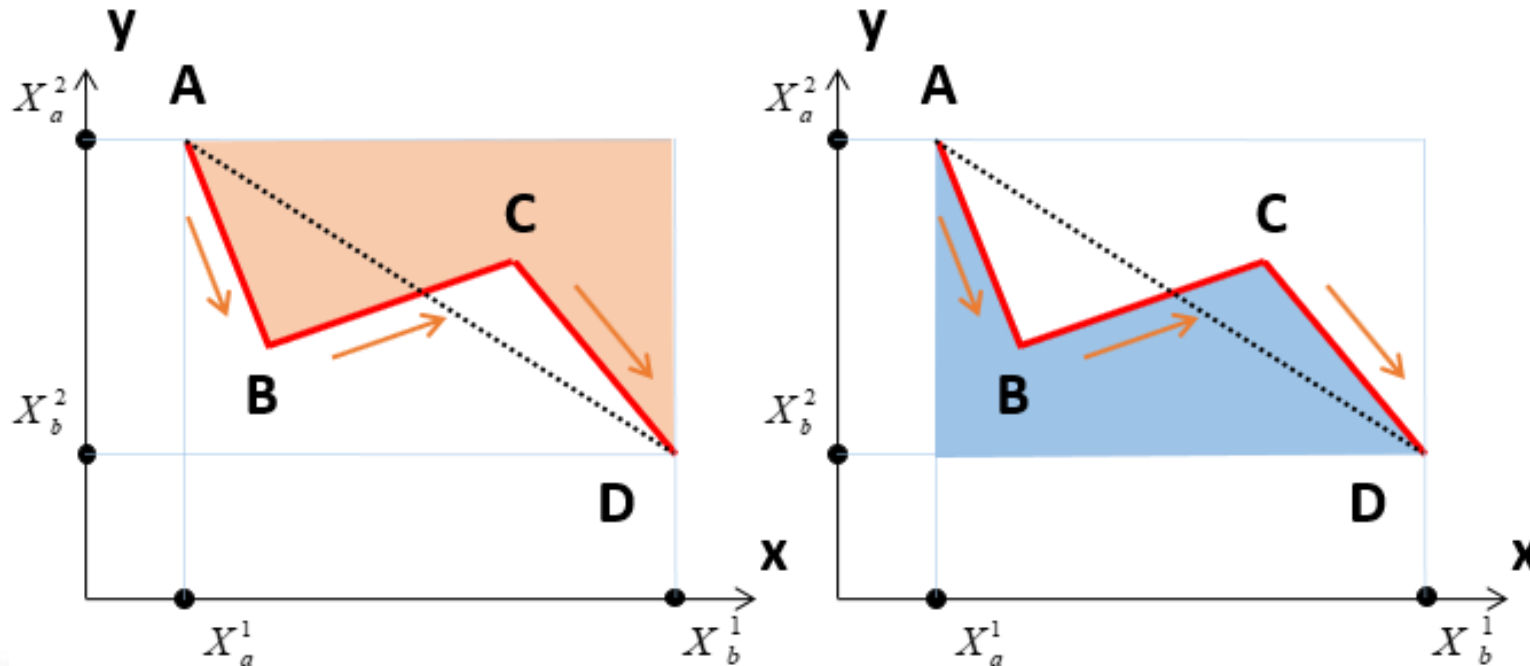
$$\begin{aligned} S(X)_{a,b}^{2,2} &= \int_{a < t_1 < t_2 < b} dX_{t_1}^2 dX_{t_2}^2 \\ &= \frac{1}{2} (X_b^2 - X_a^2)^2 \end{aligned}$$



Path signature in 2D

- The 2-fold terms (Areas)

$$S(X)_{a,b} = (1, S(X)_{a,b}^1, S(X)_{a,b}^2, S(X)_{a,b}^{1,1}, \boxed{S(X)_{a,b}^{1,2}, S(X)_{a,b}^{2,1}}, S(X)_{a,b}^{2,2}, \dots)$$

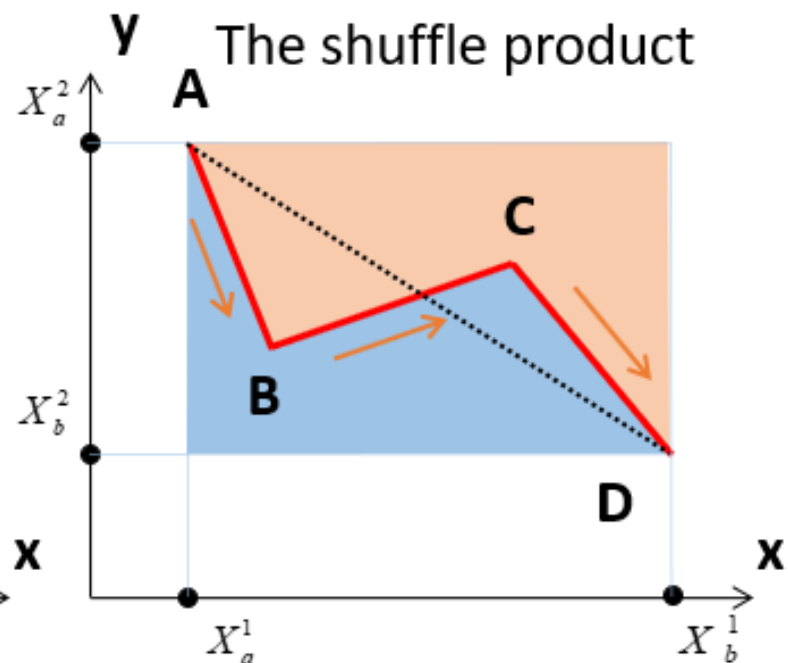
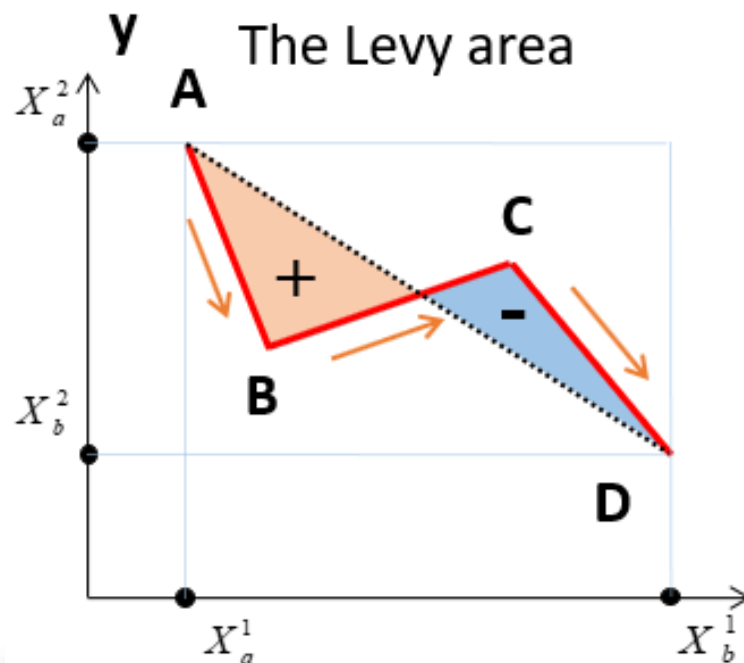


$$S(X)_{a,b}^{1,2} = \int_{a < t_1 < t_2 < b} dX_{t_1}^1 dX_{t_2}^2 \quad S(X)_{a,b}^{2,1} = \int_{a < t_1 < t_2 < b} dX_{t_1}^2 dX_{t_2}^1$$

Path signature in 2D

- The 2-fold terms (Areas)

$$S(X)_{a,b} = (1, S(X)_{a,b}^1, S(X)_{a,b}^2, S(X)_{a,b}^{1,1}, \boxed{S(X)_{a,b}^{1,2}, S(X)_{a,b}^{2,1}}, S(X)_{a,b}^{2,2}, \dots)$$



$$A = S(X)_{a,b}^{1,2} - S(X)_{a,b}^{2,1}$$

$$S(X)_{a,b}^1 \cdot S(X)_{a,b}^2 = S(X)_{a,b}^{1,2} + S(X)_{a,b}^{2,1}$$

Example of path signature

$$X_t = \{X_t^1, X_t^2\} = \{3+t, (3+t)^2\} \quad t \in [0, 5], \quad (a=0, b=5),$$

$$dX_t = \{dX_t^1, dX_t^2\} = \{dt, 2(3+t)dt\}.$$

$$S(X)_{0,5}^{1,1} = \int_{0 < t < 5} dX_t^1 = \int_0^5 dt = X_5^1 - X_0^1 = 5,$$

$$S(X)_{0,5}^{2,2} = \int_{0 < t < 5} dX_t^2 = \int_0^5 2(3+t) dt = X_5^2 - X_0^2 = 55,$$

$$S(X)_{0,5}^{1,1} = \iint_{0 < t_1 < t_2 < 5} dX_{t_1}^1 dX_{t_2}^1 = \int_0^5 \left[\int_0^{t_2} dt_1 \right] dt_2 = \frac{25}{2},$$

$$S(X)_{0,5}^{1,2} = \iint_{0 < t_1 < t_2 < 5} dX_{t_1}^1 dX_{t_2}^2 = \int_0^5 \left[\int_0^{t_2} dt_1 \right] 2(3+t_2) dt_2 = \frac{475}{3},$$

$$S(X)_{0,5}^{2,1} = \iint_{0 < t_1 < t_2 < 5} dX_{t_1}^2 dX_{t_2}^1 = \int_0^5 \left[\int_0^{t_2} 2(3+t_1) dt_1 \right] dt_2 = \frac{350}{3},$$

$$S(X)_{0,5}^{2,2} = \iint_{0 < t_1 < t_2 < 5} dX_{t_1}^2 dX_{t_2}^2 = \int_0^5 \left[\int_0^{t_2} 2(3+t_1) dt_1 \right] 2(3+t_2) dt_2 = \frac{3025}{2},$$

$$S(X)_{0,5}^{1,1,1} = \iiint_{0 < t_1 < t_2 < t_3 < 5} dX_{t_1}^1 dX_{t_2}^1 dX_{t_3}^1 = \int_0^5 \left[\int_0^{t_3} \left[\int_0^{t_2} dt_1 \right] dt_2 \right] dt_3 = \frac{125}{6},$$

$\vdots$

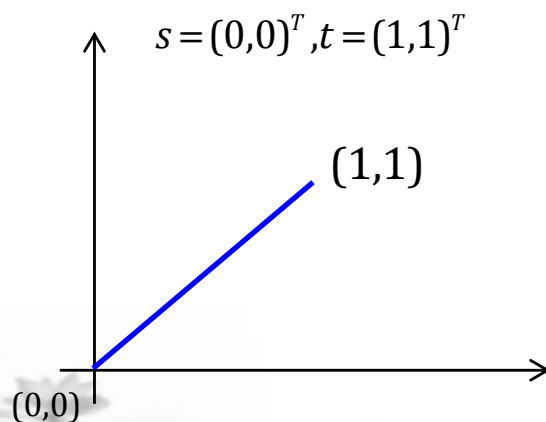


Truncated path signature

- k-th level of the path signature: defined as the finite collection of all terms $S(X)_{a,b}^{i_1, \dots, i_k}$, where the multi-index is of length k

For Example, the second level of Path Signature is given by:

$$S(X)_{a,b}^{1,1}, \dots, S(X)_{a,b}^{1,d}, S(X)_{a,b}^{2,1}, \dots, S(X)_{a,b}^{d,d}.$$



$$X_{s,t}^0 = 1 \quad X_{s,t}^1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad X_{s,t}^2 = \frac{1}{2} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{pmatrix}$$

For $m=2$,

$$S(X)_{s,t} = (1, 1, 1, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2})$$

Property of Path Signature (PS)

- 唯一性：
 - 任意一个非树状路径（tree-like path）路径均可被其Path Signature唯一地确定。
- 平移不变性、旋转不变形
 - PS与路径始位置点无关；某些阶的PS及其组合具有旋转不变形
- 时间重参数化不变性、不同时间跨度下的不变形
 - 在时间重参数化（eg 采样率）下的不变性
 - 意义：PS对采样率大小、丢失采样点、或添加随机噪声等不太敏感
- 洗牌积等式（shuffle product identity）等效性

$$S(P)_{0,T}^I S(P)_{0,T}^J = \sum_{K \in R(I,J)} S(P)_{0,T}^K.$$
$$S(P)_{0,T}^1 \cdot S(P)_{0,T}^2 = S(P)_{0,T}^{12} + S(P)_{0,T}^{21}.$$

- 意义：将PS作为特征时，低阶非线性可用高阶线性组合等效表示（
eg 低阶线性不可分可转换为高阶线性可分问题来求解）

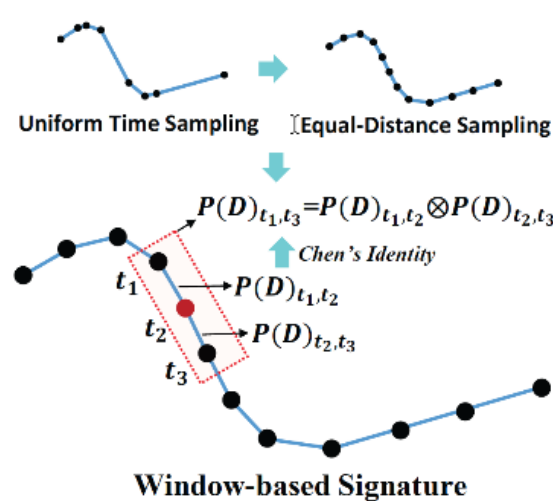
实际应用举例：提取手写汉字的Path Signature 特征表达

- An online handwritten character consists of strokes
- Let $X_i(t)$ denote the i th point on a stroke at time t , the corresponding m level signature can be computed by:

$$S(P)_{i-\delta, i+\delta}^m = (1, P_{i-\delta, i+\delta}^1, P_{i-\delta, i+\delta}^2, \dots, P_{i-\delta, i+\delta}^m)$$

where δ is an interval window parameter (eg. = 3~8)

- Repeat the above procedure for every point on the stroke, stroke by stroke



$$P(D)_{t_1, t_2} = \begin{pmatrix} 1, 1, -1, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2} \end{pmatrix}$$

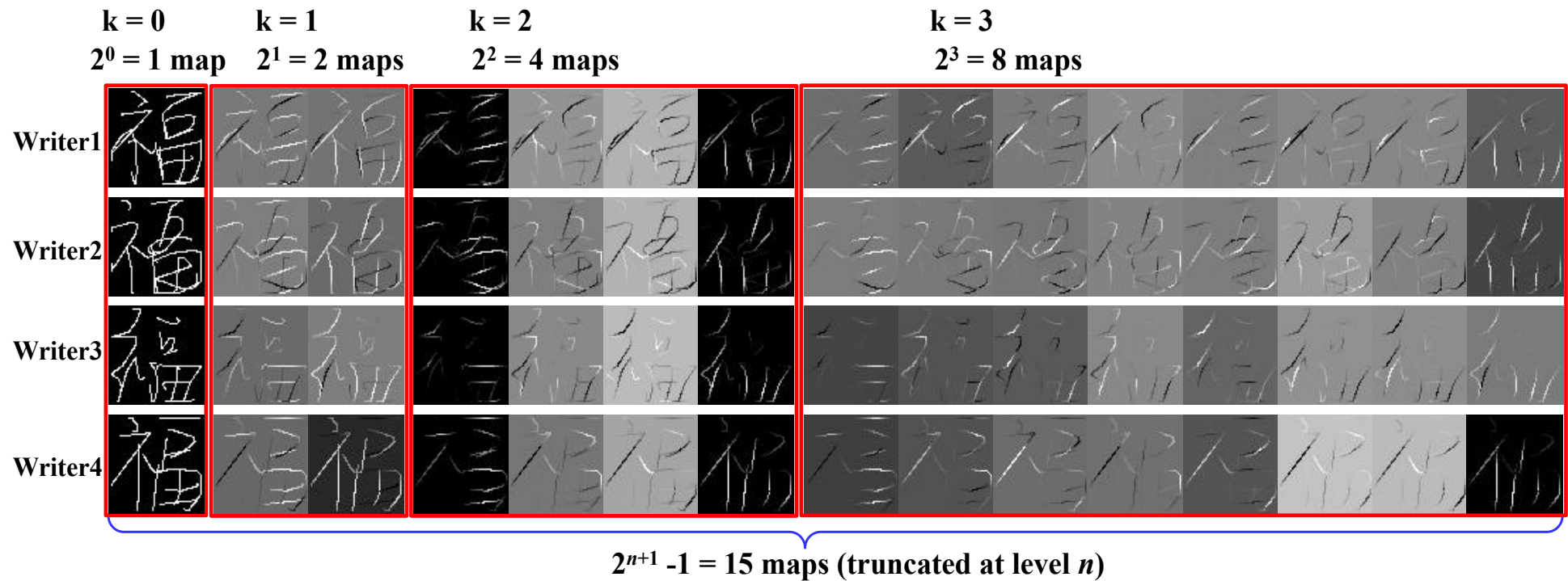
Eq. (5)

$$P(D)_{t_1, t_2}^0 = 1, P(D)_{t_1, t_2}^1 = \frac{1 \otimes \begin{bmatrix} 1 \\ -1 \end{bmatrix}}{1} = \begin{bmatrix} 1 \\ -1 \end{bmatrix},$$

$$P(D)_{t_1, t_2}^2 = \frac{\begin{bmatrix} 1 \\ -1 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ -1 \end{bmatrix}}{2} = \begin{bmatrix} 1/2 \\ -1/2 \\ -1/2 \\ 1/2 \end{bmatrix}.$$

Eq. (2)

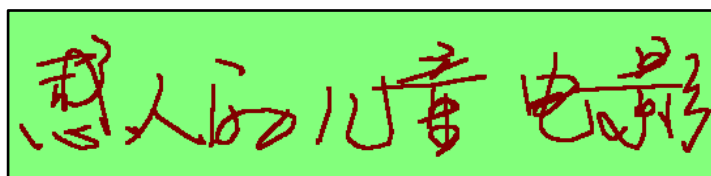
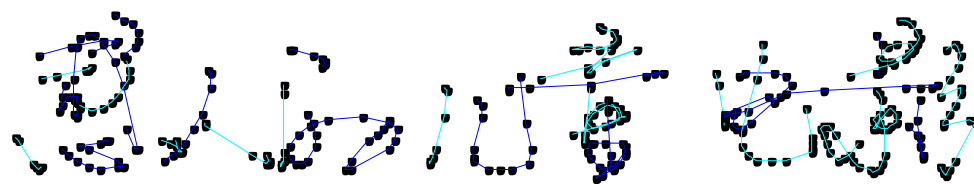
Visualization of Path Signature Maps of Chinese Characters



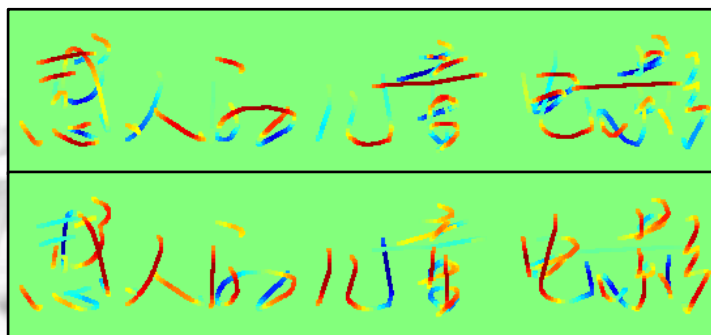
level	# of each level signature maps	# of truncated signature maps	Representation remarks
0	1	1	Bitmap
1	2	3	Gradient direction information
2	4	7	Area
k	2^k	$2^{k+1}-1$	?

More Visualization of Path Signature of Chinese Handwritten Text

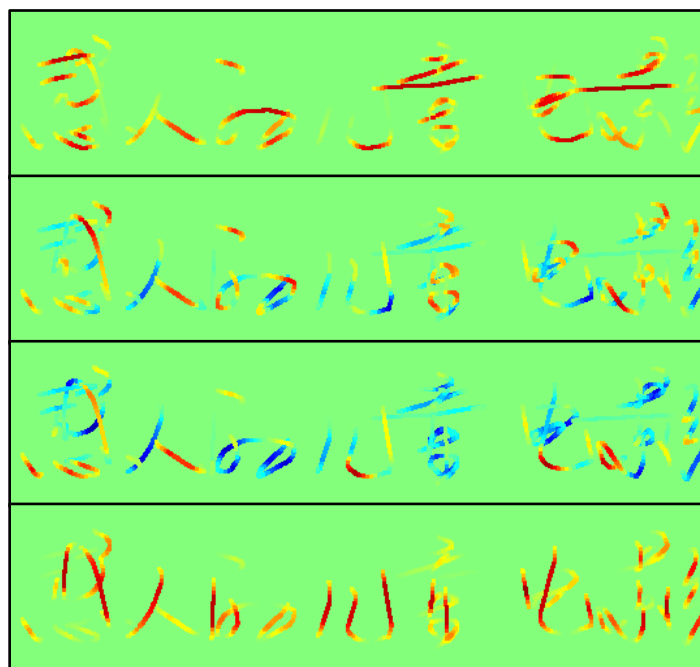
□ Heat map of path signature feature



$k = 0$



$k = 1$



$k = 2$

More Visualization of Path Signature of English Handwritten Text

Ryder Cup World Points

Ryder Cup World Points

$k = 0$

Ryder Cup World Points
Ryder Cup World Points

$k = 1$

Ryder Cup World Points
Ryder Cup World Points
Ryder Cup World Points
Ryder Cup World Points

$k = 2$

Ryder Cup World Points

Ryder Cup World Points

Ryder Cup World Points

Ryder Cup World Points

Ryder Cup World Points

Ryder Cup World Points

Ryder Cup World Points

Ryder Cup World Points

Ryder Cup World Points

$k = 3$

Path Signature for ML & HCCR

Sparse arrays of signatures for online character recognition

Benjamin Graham
University of Warwick
b.graham@warwick.ac.uk

November 12, 2013

A Primer on the Signature Method in Machine Learning

Ilya Chevyrev^a and Andrey Kormilitzin^{a,b}

^aMathematical Institute, University of Oxford, Andrew Wiles Building, Woodstock Road, Oxford, OX2 6CG, UK

^bOxford-Man Institute, University of Oxford, Eagle House, Walton Well Road, Oxford, OX2 6ED, UK

E-mail: ilya.chevyrev@maths.ox.ac.uk,
andrey.kormilitzin@maths.ox.ac.uk

2016

- **Results**

- ICDAR 2013: 97.39% (**Winner of ICDAR 2013 OHCCR competition**)
- CASIA-OLHWDB 1.1: 96.62%
- MNIST: 99.69%
- CIFAR 10: 95.53% (**Kaggle CIFAR 10/100 winner in 2014**)

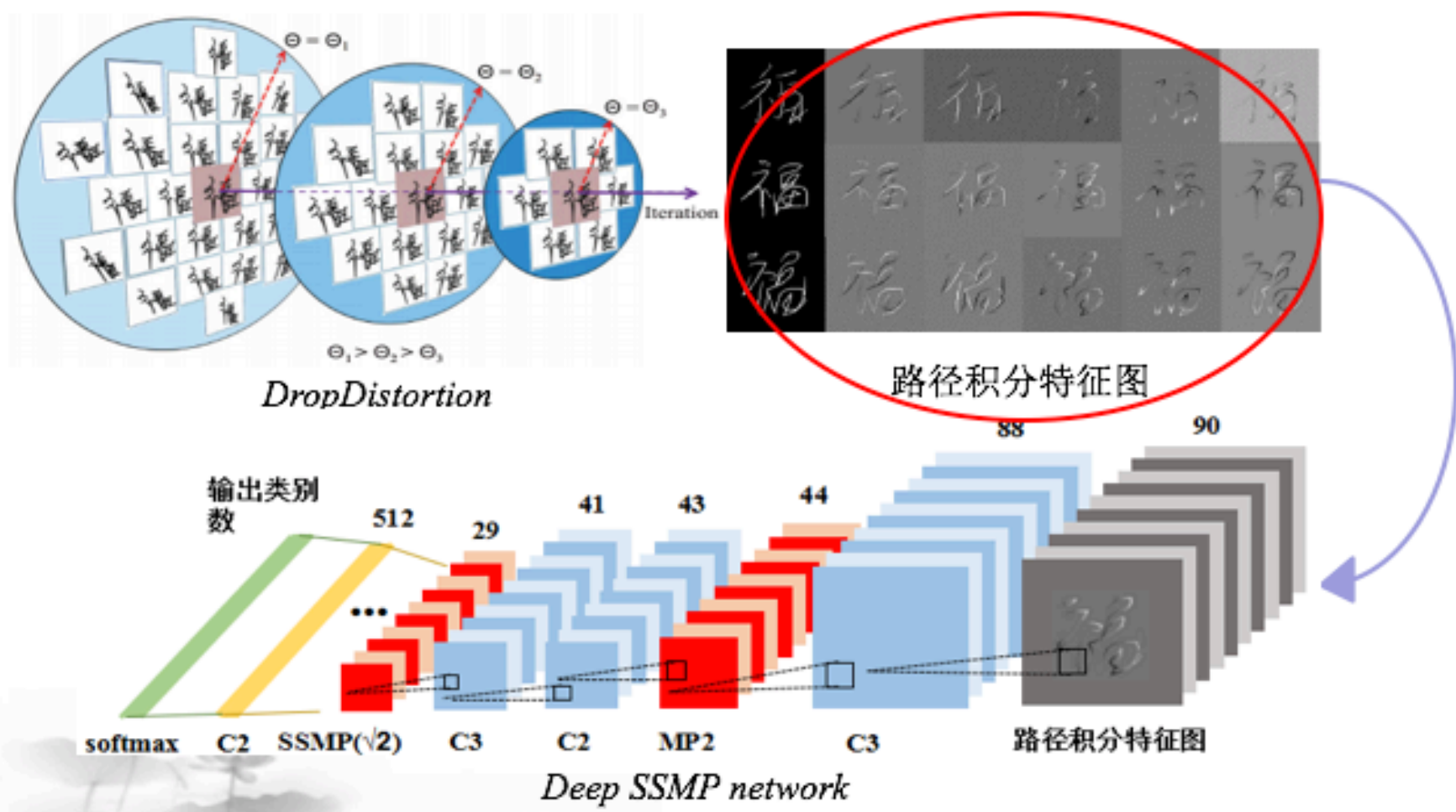
Path Signature open source tools:

- Esig (CoRoPa) Library: <https://coropa.sourceforge.io>
- iisignature package: <https://github.com/bottler/iisignature>

Rotation Free & Large Vocabulary Online Chinese Character Recognition

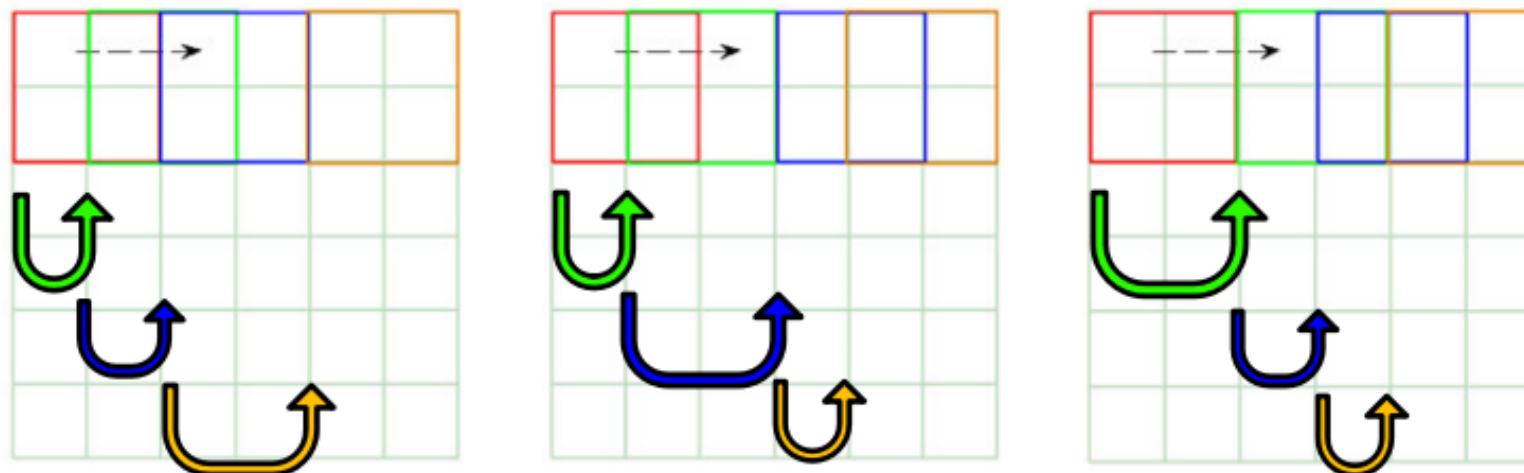


High Performance Large Scale OHCCR with CNN and Path Signature



- WX Yang, LW Jin, et al, [ICDAR 2015](#)
- WX Yang, LW Jin, et al, [Pattern Recognition 2016](#)
- SX Lai, LW Jin, WX Yang, [Pattern Recognition Letters, 2017](#)

Fractional max-pooling



Fractional max-pooling (FMP):

- Stochastic pooling strides.
- Coupled with character distortion
- Can perform multiple tests, which is similar to models ensembling, to improve the performance without any additional storage requirement.

B. Graham, [arXiv 2014](#).

S. Lai, LW. Jin, W. Yang. *Pattern Recognition Letters*, 2017.

Experiments of OHCCR

Experimental Data

– CASIA-OLHWDB 1.1

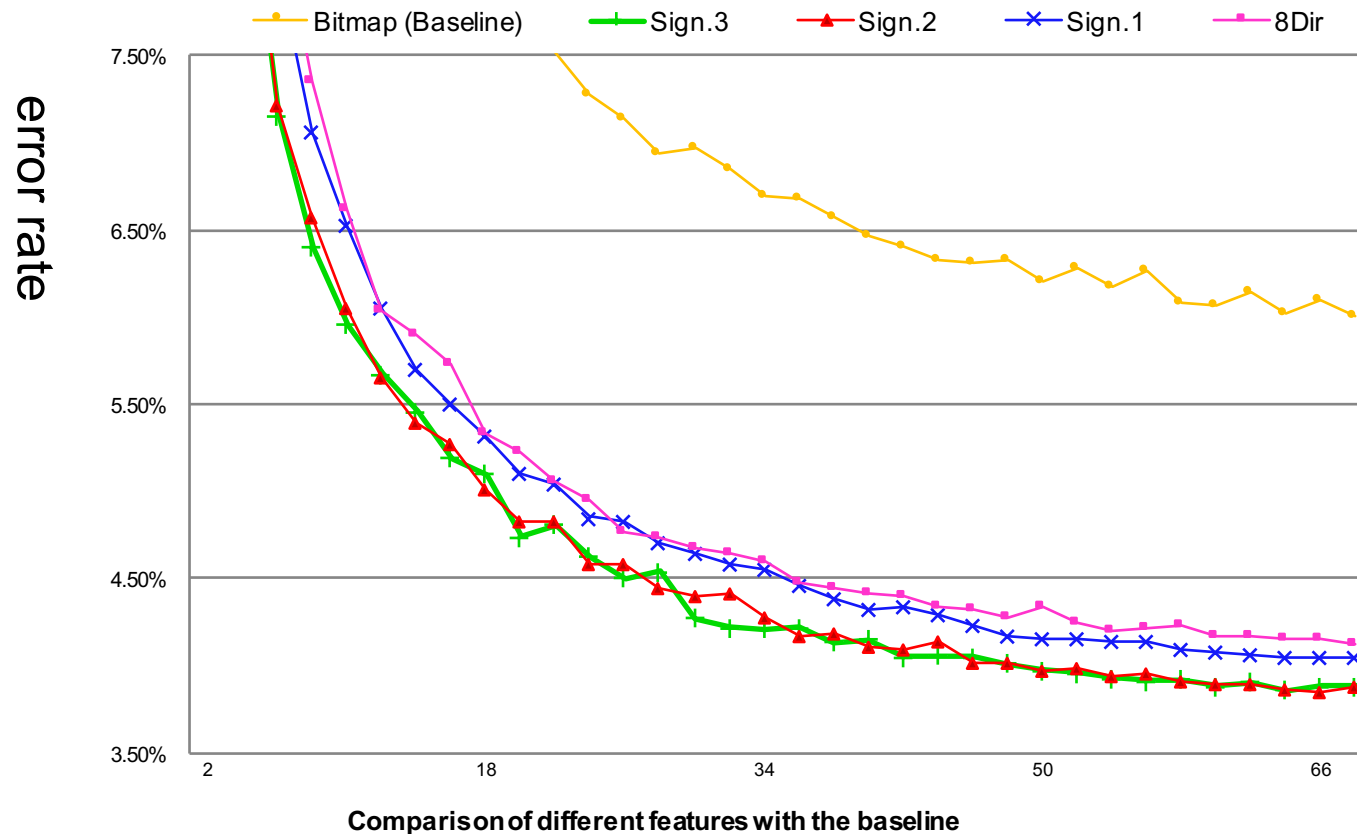
- 3755 classes, 300 writers,
- Training data: $3755 * 240 = 901,200$
- Test data: $3755 * 60 = 225,300$

– ICDAR 2013 Competition DB

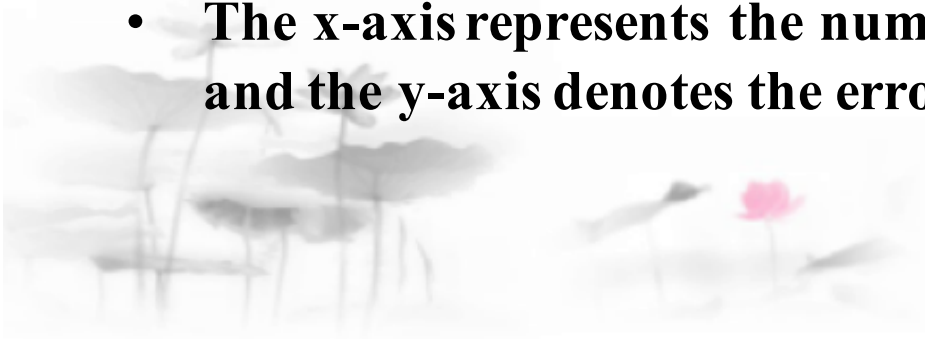
- $3755 * 60 = 225,300$ samples (used as testing data)



Comparison of different feature representations



- The x-axis represents the number ($\times 10^4$) of mini-batches for training and the y-axis denotes the error rate for testing data.



Comparison with previous methods

Methods	DB 1.0	DB 1.1	ICDAR2013 Competition	Using PS/Dir?
Best Traditional	95.28	94.85	95.31	No
Ensemble of CNNs (CVPR 2012)	94.39	--	--	No
DeepCNet (Graham, TR 2013)	—	96.42	97.37	Yes
FMP ensemble (Graham, arXiv 2014)	—	97.03	—	Yes
DropSample+PS+CNN (ours , PR 2016)	97.33	97.06	97.51	Yes
DirectMap+CNN (Zhang et al., PR 2017)	—	—	97.55	Yes
FMP-CNN+PS (ours , PRL 2017)	97.67	97.30	97.99	Yes

Experimental result on very large scale dataset

- Very Large Scale HCCR with CNN/LSTM
 - # of classes: 27738 (covers 27533 GB18030-2000 set, English letters, and extended symbols)
 - ✓ 6763 GB2312 simplified Chinese
 - ✓ 2000+ Traditional Chinese
 - ✓ 1100+ Hong Kong characters
 - ✓ English letter
 - ✓ Digit
 - ✓ Symbols
 - Resources of data:
 - SCUT-COUCH (Jin et al, IJDAR 2011)
 - Part of CASIA OLHWDB1.0-1.02, ICDAR 2013 (Liu et al, PR 2013)
 - HKU (Bai & Huo, ICDAR 2005)
 - 863 (Qiao et al, CHTL 2005)
 - In-house data
 - Data augmentation
 - Affine transformation, deformation transformation (Jin et al 2000) et al
 - Total number of samples
 - 20,928,700 (training); 1,052,985 (testing)

Results (27738 classes)

Recognition accuracies (%) on eight testing datasets

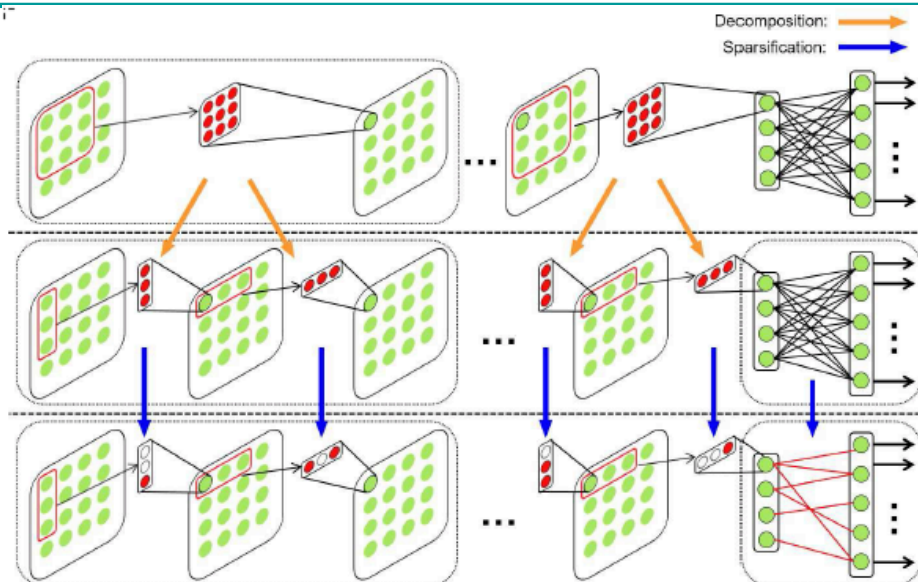
Model	Casia1.0	Casia1.1	Casia1.2	SCUT-COUCH	HKU	In-house	863	ICDAR	Average
# of samples	143600	98235	79154	161166	120863	184089	40578	225300	--
PS-CNN ^[1]	97.73	97.24	98.04	99.06	98.49	98.89	99.77	95.58	97.82
LSTM ^{[2]*}	97.87	97.35	98.26	98.67	98.36	98.82	99.59	96.04	97.87

* Our LSTM model is inspired from [2] with the following architecture:
Input-128LSTM-512LSTM-512FC-27738 softmax - output

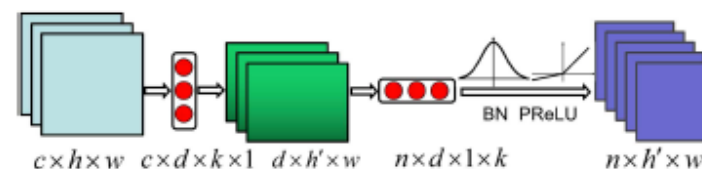
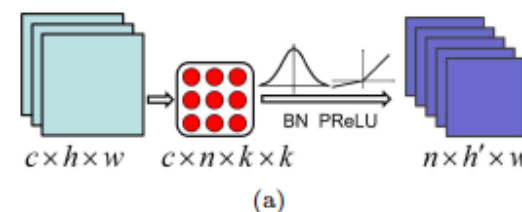
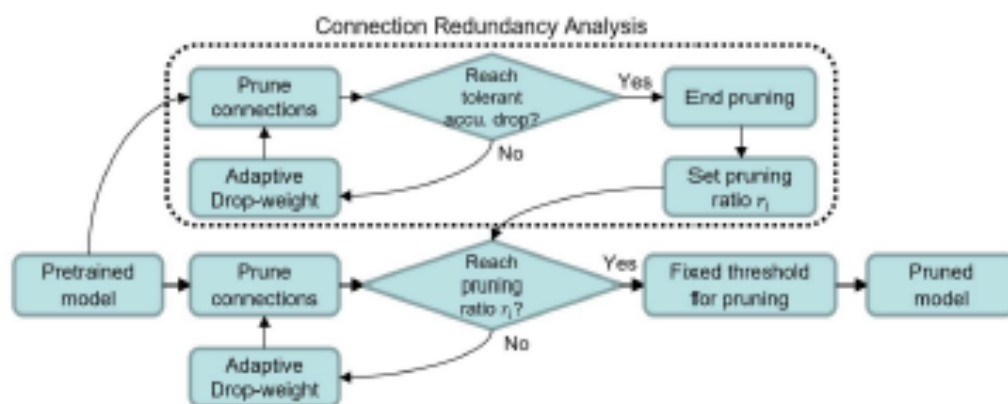
[1] S.X. Lai, L.W. Jin, W.X. Yang, Pattern Recognition Letters, 2017 .

[2] X.Y. Zhang, F. Yin, C.L. Liu, et al, IEEE TPAMI 2018

Building Compact and Fast CNN for HCCR



- **Low-Rank Matrix Decomposition**
- **Connections Pruning with Adaptive Drop-Weight**
- **BLAS library**
- **Loop Unrolling**
- ...

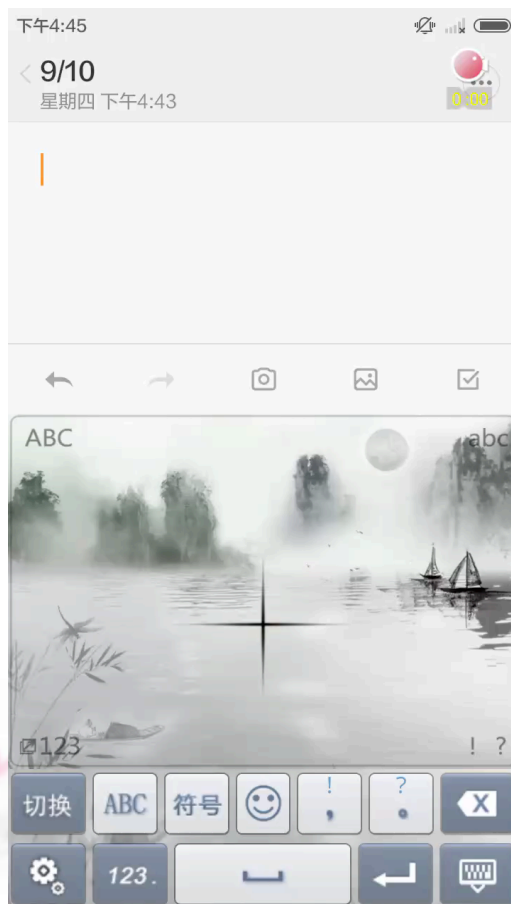
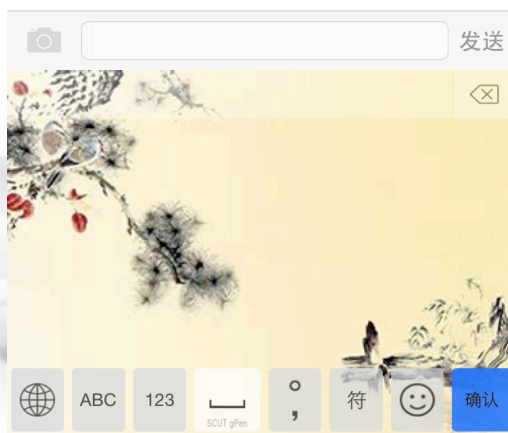


• **Code:** <https://github.com/HCIILAB/Forward-Implementation-of-Fast-and-Compact-CNN-for-Offline-HCCR>

XF Xiao, LW Jin, et al, *Pattern Recognition*, 2017

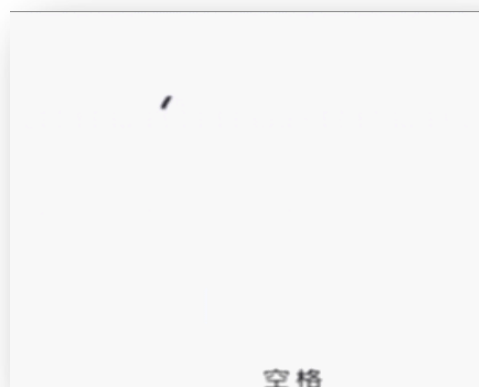
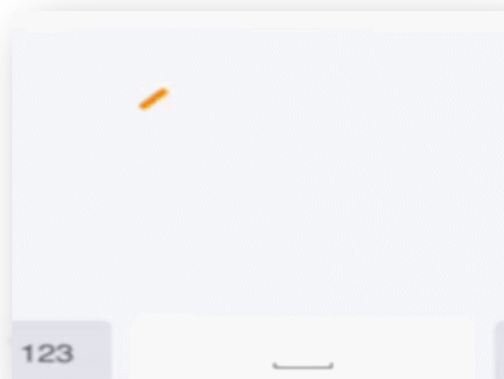
基于深度学习的SCUT gPen手写输入法

- 支持简体、繁体、香港字、字母、标点、数学符号等**10000**余个类别的手写识别
- 针对CPU运算进行了优化，无需GPU支持，识别速度快！
 - ✧ **Android手机平台：23~70ms/char**
 - ✧ **iOS平台：15~50ms/char**
 - ✧ **PC平台：6~27ms/char**

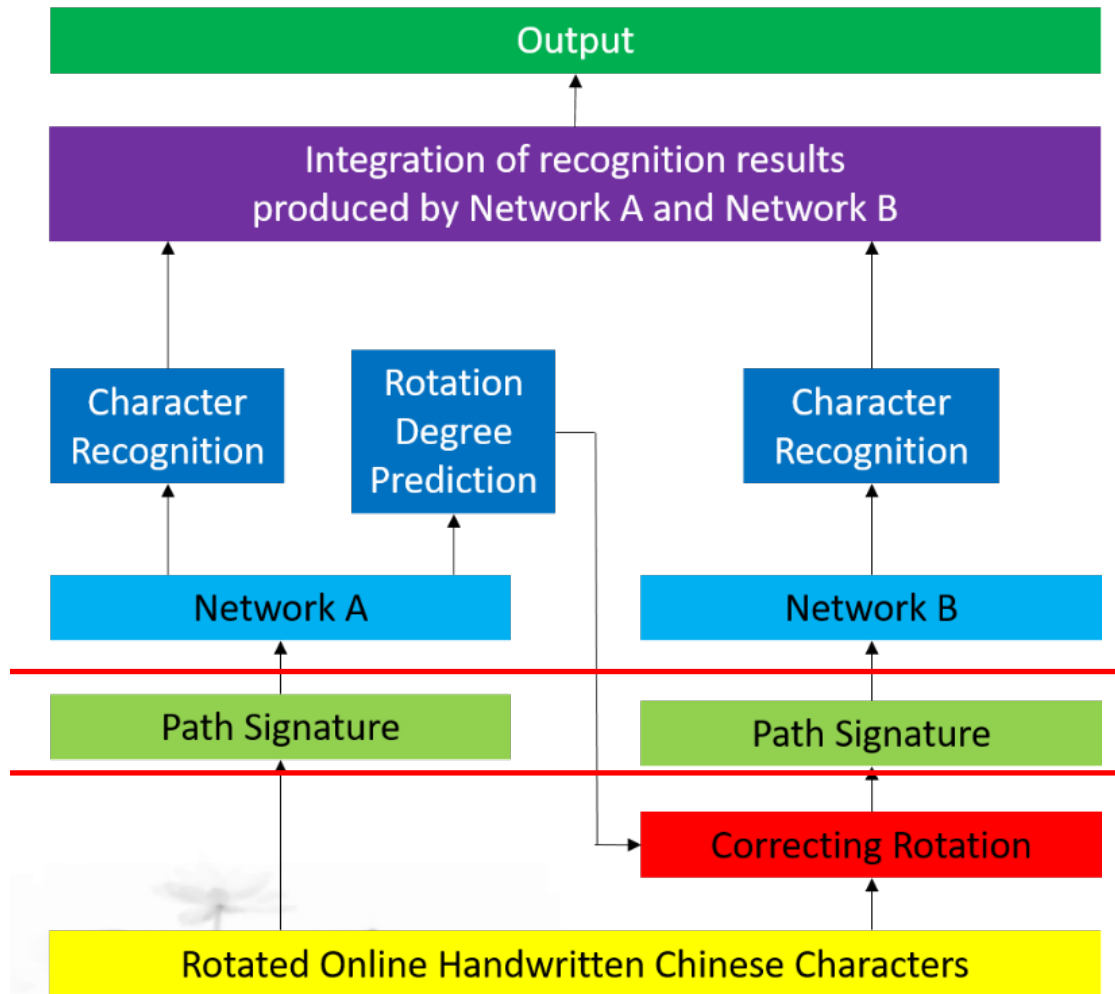


Rotation Free OLHCCR: Motivation

- It is common that people sometime write charater with a slight rotation degree, especially on mobile devices
- China national standard (**GB/T18790-2010**) requires the HCCR software should be able to recognize a character with a rotation angles of up to $\pm 45^\circ$
- The recognition accuracy will decrease significantly if the rotation degree is bigger than a certain degree (eg $\pm 30^\circ$)



A two-stage CNN for rotation free OLHCCR



Layer	Network A	Network B
Convolution	$3 \times 3 \times 64$	
Max Pooling	3×3	
Convolution	$3 \times 3 \times 96$	
Max Pooling	3×3	
Convolution	$3 \times 3 \times 128$	
Max Pooling	3×3	
Convolution	$3 \times 3 \times 256$	
Convolution	$3 \times 3 \times 256$	
Max Pooling	3×3	
Convolution	$3 \times 3 \times 384$	
Convolution	$3 \times 3 \times 384$	
Max Pooling	3×3	
Fully Connected	128	
Dropout	0	0.3
Fully Connected	1	3755
Loss Function	Euclidean	Softmax

Z Li, LW Jin, SX Lai, ICFHR 2018

Experiments and results

- **Dataset**

- **Training: CASIA OLHWDB 1.0&1.1**
- **Testing: ICDAR 2013 (with random rotation degree of $\pm 45^\circ$)**

Effectiveness of Path Signature (Network A, $\pm 45^\circ$)

Path Signature	Degree prediction accuracy	Recognition accuracy
X	90.99	93.91
✓	91.89	96.41

Performahce of Network A + Network B ($\pm 45^\circ$)

Model	Recognition accuracy
Network A	96.41
Network B	97.21
Network A+B	97.38

Experimental results (cont.)

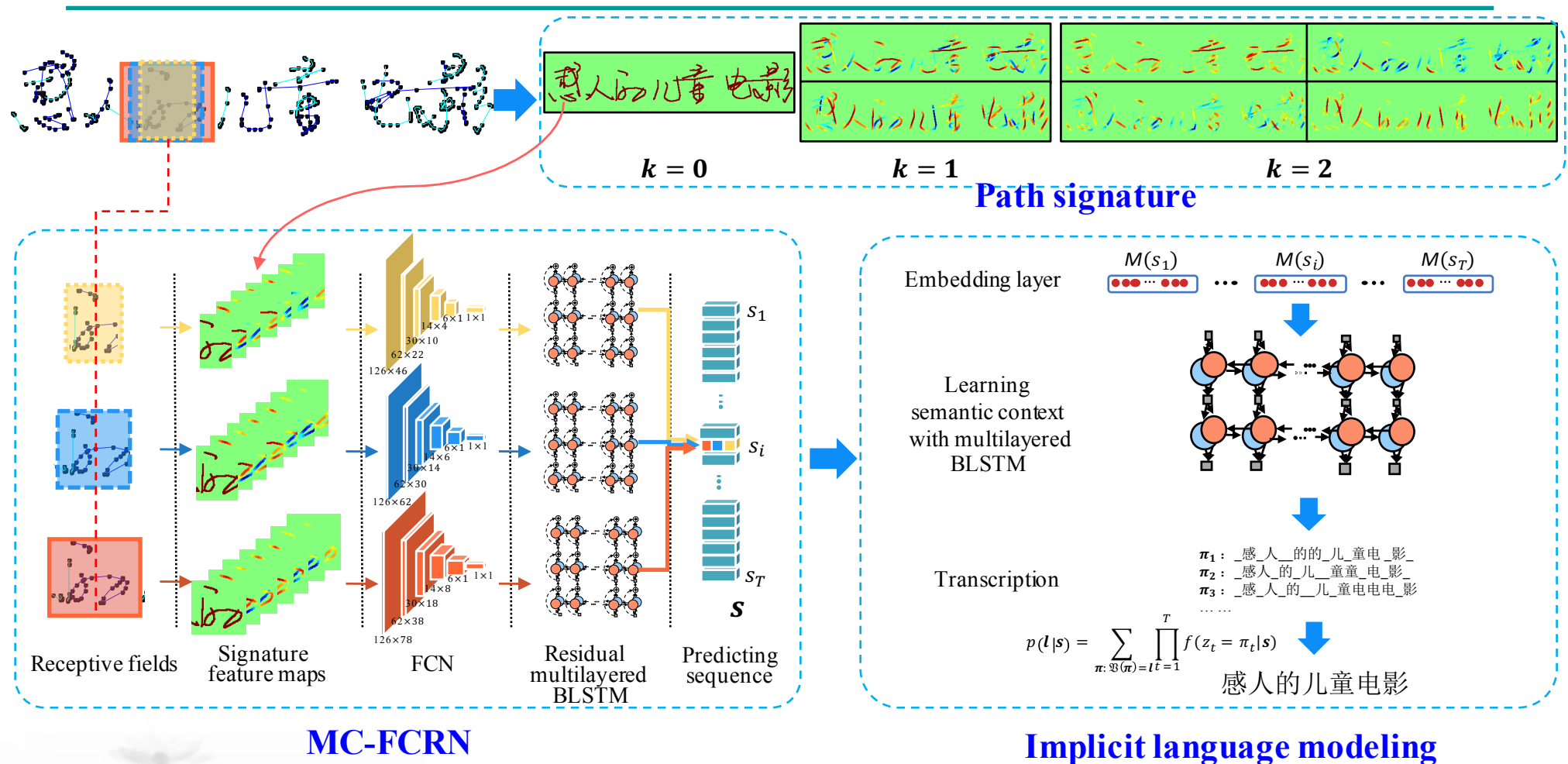
- Comparison on regular dataset with SOTA methods
 - All the data are not rotated

Method	Rotation Free?	Accuracy (%)
DEF+DLQDF (ICDAR 2013)	X	95.31
DropSample (PR 2016)	X	97.23
HCCR-Inc-GAP (ICDAR 2017)	X	97.45
DirectMap+CNN (PR 2017)	X	97.64
DropDistortion (PRL 2017)	X	97.79
RNN (TPAMI 2018)	X	97.89
Our (two stage CNN)	X	97.38
	✓ ($\pm 45^\circ$)	97.38
	X	97.18
	✓ ($\pm 180^\circ$)	97.09

Deep Learning for Handwritten Text Line Recognition

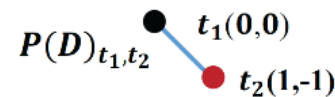
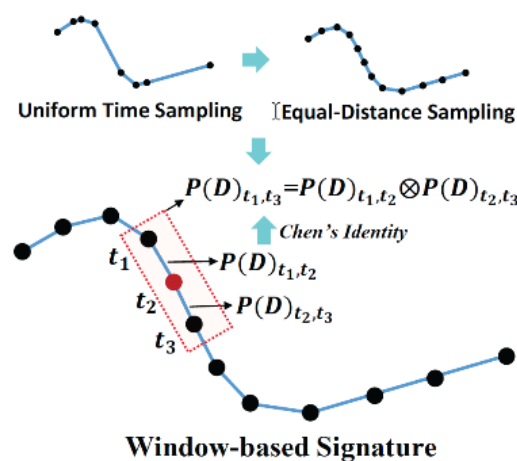


OHCTR with Fully Convolutional Recurrent Network



ZC Xie, ZH Shun, LW Jin, H Ni, T Lyons, Learning Spatial-Semantic Context with Fully Convolutional Recurrent Network for Online Handwritten Chinese Text Recognition, **IEEE TPAMI**, 2018.

Path Signature Feature Maps



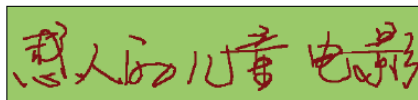
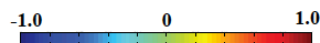
Eq. (5)

$$P(D)_{t_1,t_2}^0 = 1, P(D)_{t_1,t_2}^1 = \frac{1 \otimes \begin{bmatrix} 1 \\ -1 \end{bmatrix}}{1} = \begin{bmatrix} 1 \\ -1 \end{bmatrix},$$

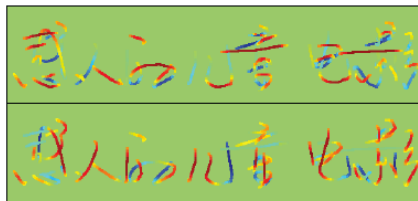
$$P(D)_{t_1,t_2}^2 = \frac{\begin{bmatrix} 1 \\ -1 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ -1 \end{bmatrix}}{2} = \begin{bmatrix} 1/2 \\ -1/2 \\ -1/2 \\ 1/2 \end{bmatrix}.$$

Eq. (2)

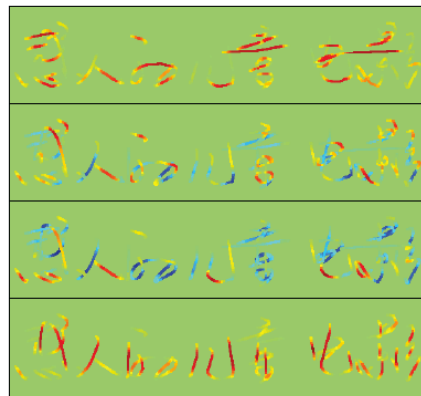
$$P(D)_{t_1,t_2} = \left(1, 1, -1, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2} \right).$$



$k = 0$

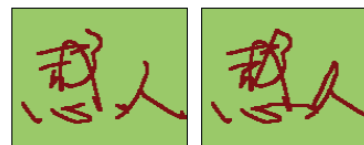
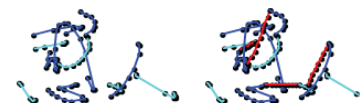


$k = 1$

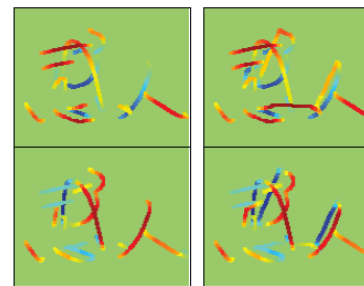


$k = 2$

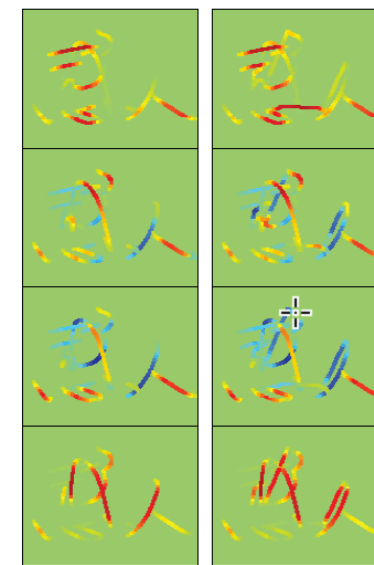
(c)



$k = 0$



$k = 1$

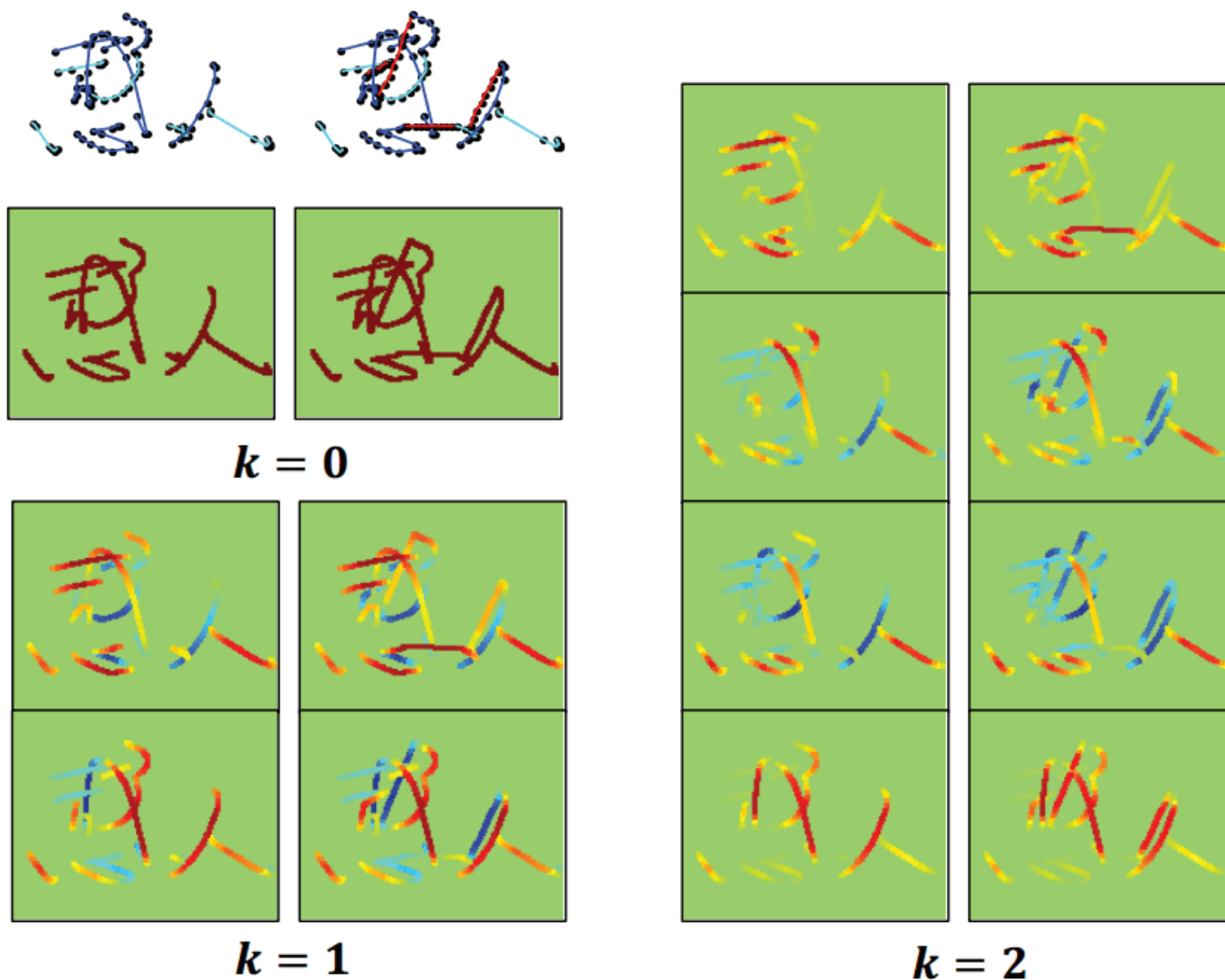


$k = 2$

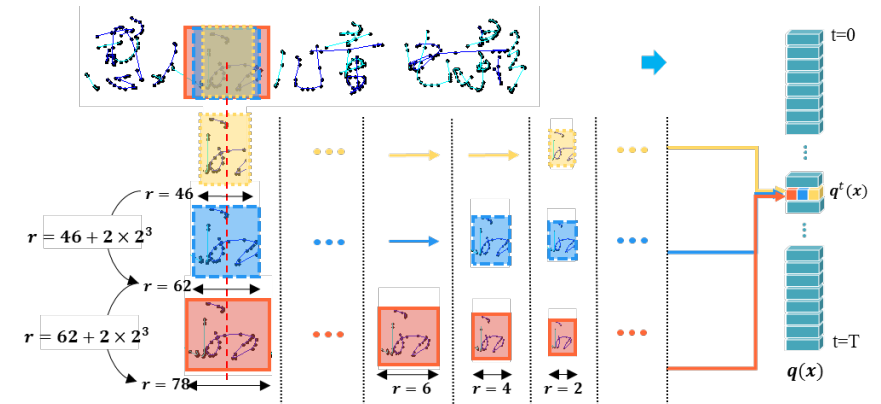
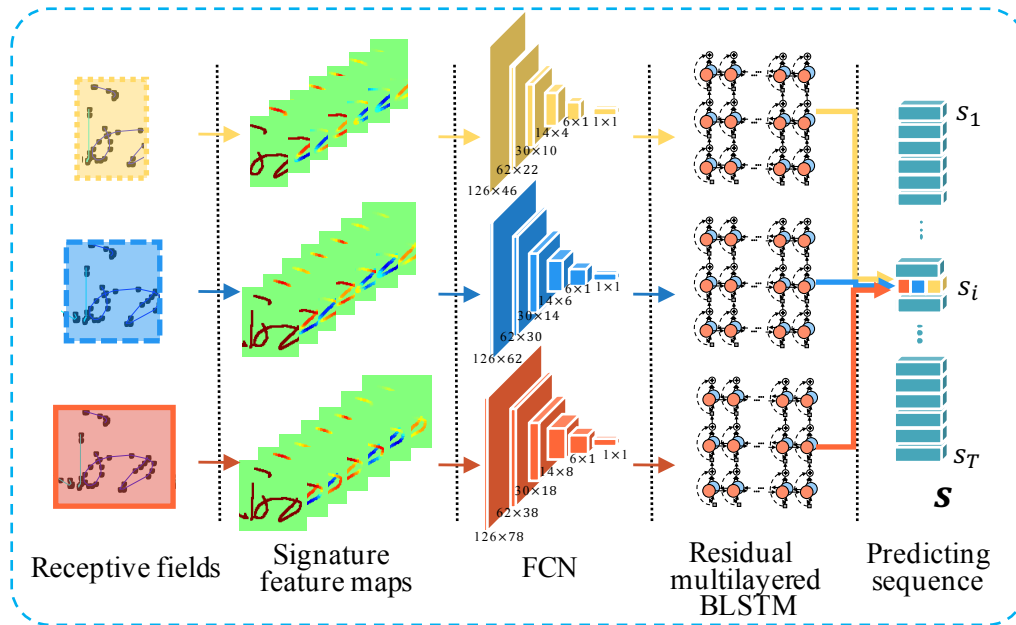
(d)

Benefit of Path Signature Representation

□ Robust against additional stroke connections



Multi-Spatial-Context FCRN (MC-FCRN)

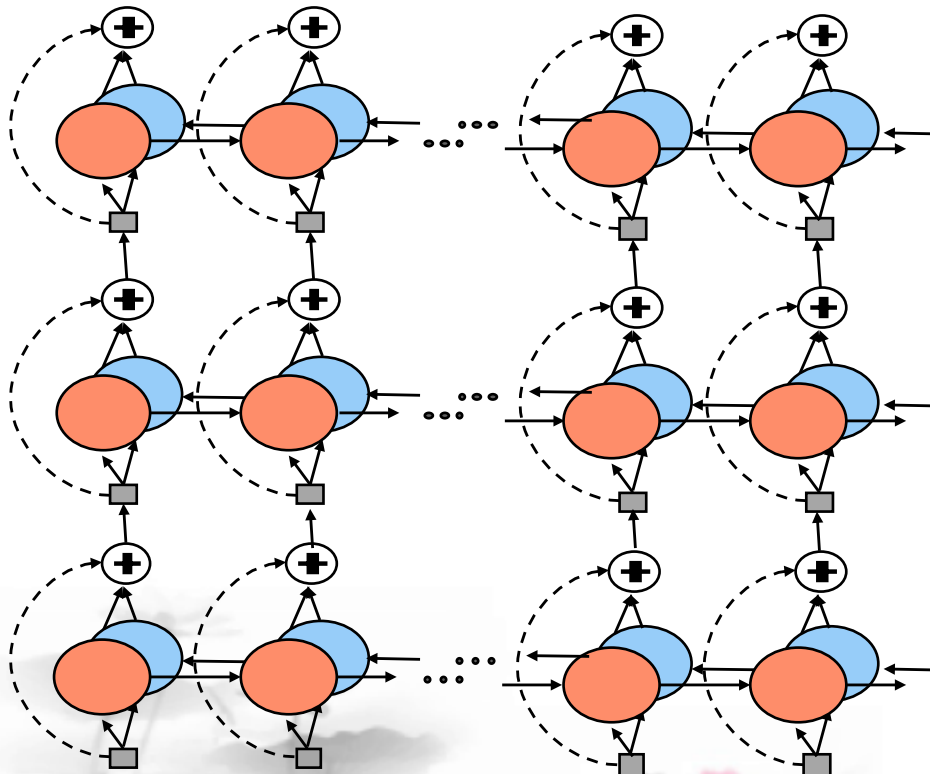


- **Multi Contextual Fully Convolutional Recurrent Networks**
 - MC-FCRN is a **segmentation-free** method that effectively captures the variable spatial contextual dynamics;
 - the main components of MC-FCRN, FCN, residual BLSTM, and CTC can be **jointly trained** to benefit from one another, thereby enhancing the overall performance

Multi-Spatial-Context FCRN

- **Residual Deep BLSTM**

- Allowing for faster training and convergence
- Enhanced performance



Effect of Residual Recurrent Network (Percent)

Architecture	Dataset-ICDAR		Dataset-CASIA	
	CR	AR	CR	AR
LSTM	85.67	84.60	90.14	89.17
Residual LSTM	87.82	86.85	92.29	91.38
BLSTM	87.96	87.28	92.69	92.15
Residual BLSTM	89.24	88.32	93.66	92.96

Connectionist Temporal Classification (CTC)

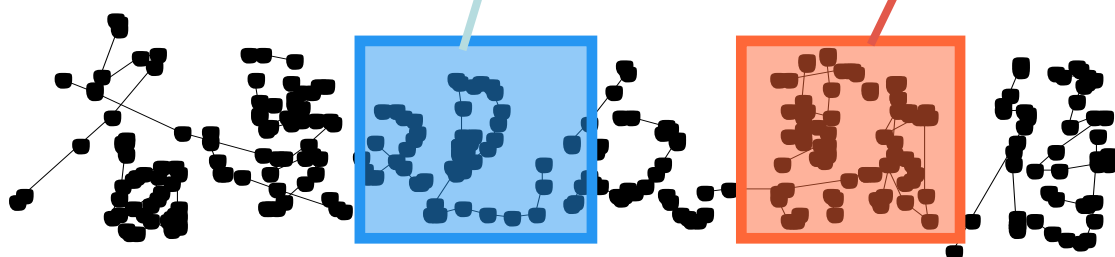
'blank'	...	0.907	0.349	...	0.1	0.82	...	0.02	...	...
观	...	0.001	0.001	...	0.789	0.1	...	0.003	...	...
...	...	0.003	0.003	...	0.08	0.007	...	0.004	...	...
...	...	.	.	...	.	.	...	.	...	...
期	...	0.002	0.001	...	0.001	0.001	...	0.001	...	...
...	...	.	.	...	.	.	...	.	...	...
...	...	0.001	0.0015	...	0.002	0.002	...	0.001	...	...

π : _备_受_观_众_期_期_待_
 π : _备_受_观_众_期_待_
 π : _备_受_观_众_期_期_期_待_
 π : _备_受_观_众_期_期_期_待_

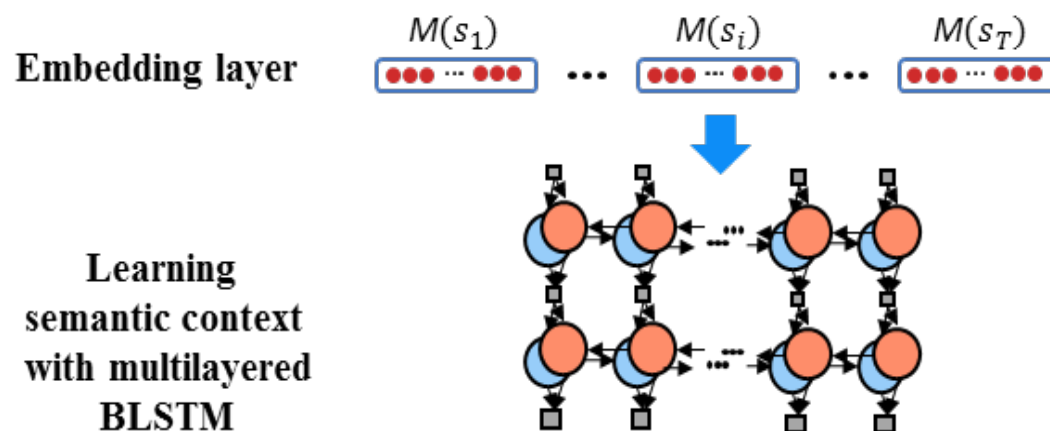
$\mathcal{B}$

备受观众期待

$$Pr(l|s) = \sum_{\pi: \mathcal{B}(\pi)=l} Pr(\pi|s)$$



Implicit Language Model



Transcription

π_1 : 感人的儿童电影
 π_2 : 感人的儿童电影
 π_3 : 感人的儿童电影

$$p(l|s) = \sum_{\pi: \mathcal{B}(\pi)=l} \prod_{t=1}^T f(z_t = \pi_t | s)$$

感人的儿童电影

Implicit language modeling

- It offers the unique advantage of **using semantic context from both directions of the text**, outperforming language models that only predict the conditional probability of the next word given previous words in one direction
- The implicit language model has **significant advantage over statistical LM in both decoding speed and prediction accuracy.**
- Unlike statistical LM whose RAM size increase as the corpus grows, **the implicit LM has a fixed size that is only determined by its network architecture.**

Experiments

- **Dataset**
 - **Training set of CASIA2.0-2.2**
 - 4072 pages of handwritten texts,
 - 41,710 text lines, including 1,082,220 characters of 2650 classes.
 - **Testing set**
 - **The testing set of CASIA2.0-2.2**
 - 1020 text pages, including 268,924 characters of 2626 classes.
 - **The test set of the ICDAR 2013 competition dataset (Dataset-ICDAR)**
 - 3432 text lines, including 91,576 characters of 1375 classes



Experiments

Effect of Semantic Context (Percent)

System		Foot print MB (RAM)	Dataset-ICDAR			Dataset-CASIA		
			runtime	CR	AR	runtime	CR	AR
FCRN(Baseline)	w.o. LM	49.8	1.8min	87.82	86.85	2.7min	92.29	91.38
implicit LM	PH	65.4	1.7 ±0.5min	93.92	93.34	5 ±0.5min	94.26	93.74
	PFR			94.21	93.69		94.43	93.89
	CLDC			95.33	94.52		96.39	95.80
statistical LM	PH	11.1	130 ±5min	91.78	90.70	360 ±10min	94.22	93.11
	PFR	6.6		91.93	90.88		94.27	93.10
	CLDC	576.3		92.67	91.81		96.09	95.50
implicit LM + statistical LM		641.7	132±5min	95.79	95.04	365±10min	96.81	96.27

Effect of Path Signature (Percent)

Path signatures	Feature maps	Dataset-ICDAR		Dataset-CASIA	
		CR	AR	CR	AR
Sig0	1	85.14	83.60	89.58	87.67
Sig1	3	86.83	85.82	91.47	90.54
Sig2	7	87.82	86.85	92.29	91.38
Sig3	15	87.94	87.20	92.22	91.58
8-Dir [29]	8	85.71	83.32	90.48	87.82
8-Dir+Sig0	9	87.67	86.52	92.05	91.00

$$CR = (N_t - S_e - D_e) / N_t$$

$$AR = (N_t - S_e - D_e - I_e) / N_t$$

N_t : total # of chars

S_e : # of substitution errors

D_e : # of deletion errors

I_e : # of insertion errors

CR: Correction Rate
AR: Accuracy Rate

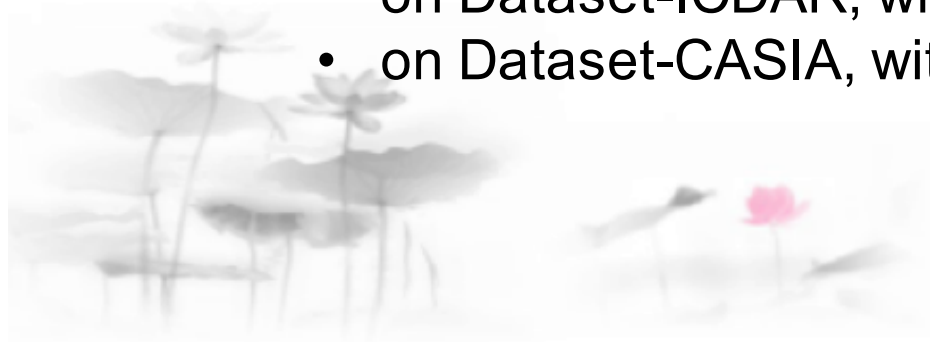
Comparison with Previous Methods

Comparison with Previous Methods Based on Correct Rate and Accuracy Rate (Percent) for Dataset-ICDAR and Dataset-CASIA

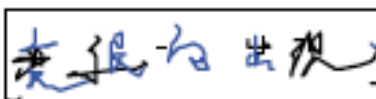
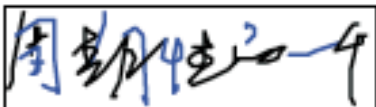
Method	Dataset-ICDAR				Dataset-CASIA			
	w.o. LM		with LM		w.o. LM		with LM	
	CR	AR	CR	AR	CR	AR	CR	AR
Shi et al., 2016 [13]	85.14	83.60	-	-	89.58	87.67	-	-
Wang et al., 2012 [1]	-	-	-	-	-	-	92.76	91.97
Zhou et al., 2013 [3]	-	-	94.62	94.06	87.93	85.92	94.34	93.75
Zhou et al., 2014 [4]	-	-	94.76	94.22	-	-	95.32	94.69
VO-3 [59]	-	-	95.03	94.49	-	-	-	-
2C-FCRN (residual LSTM) + CLDC (implicit LM)	90.17	88.88	95.11	94.50	94.47	93.31	96.36	95.96
2C-FCRN (residual LSTM) + CLDC (statistical LM)	90.17	88.88	94.51	93.45	94.47	93.31	97.01	96.49
2C-FCRN (residual LSTM) + CLDC (implicit LM & statistical LM)	90.17	88.88	96.09	95.59	94.47	93.31	97.19	96.87
Sun et al., 2016 [67] (2765 classes)	90.18	89.12	94.43	93.40	95.82	95.30	97.55	97.05
3C-FCRN (2651 classes)	93.53	92.86	97.15	96.50	95.50	94.73	97.75	97.31

Upper: 7356 classes; Lower: less than 3000 classes.

- Our method outperforms previous best results
 - on Dataset-ICDAR, with relative error reduction of 20%
 - on Dataset-CASIA, with relative error reduction of 41%



Typical Results

			
Sig0:	观春告示	留童航班	无法换制
Sig3:	观看告示	留意航班	无法控制
Ground truth:	观看告示	留意航班	无法控制
			
LSTM:	土假滴水不漏	哀很的出现	自弹打比赛
Residual LSTM:	工作滴水不漏	衰退的出现	的单打比赛
Ground truth:	工作滴水不漏	衰退的出现	的单打比赛
			
FCRN:	延误信息的布	走计了华尔街	周期4生的一个
3C-FCRN:	延误信息的公布	走到了华尔街	周期性生一个
Ground truth:	延误信息的公布	走到了华尔街	周期性的一个



Application of Path Signature for handwritings analysis and identification



Writer Identification with **Path Signature** feature and **DropStroke/Segment** techniques

❑ **Writer identification (WI)**

- Identifying a writer's handwriting is a challenging problem in the AI and pattern recognition fields.
- WI requires a large amount of data to ensure a sufficient generalization capacity. (**text-independent**)

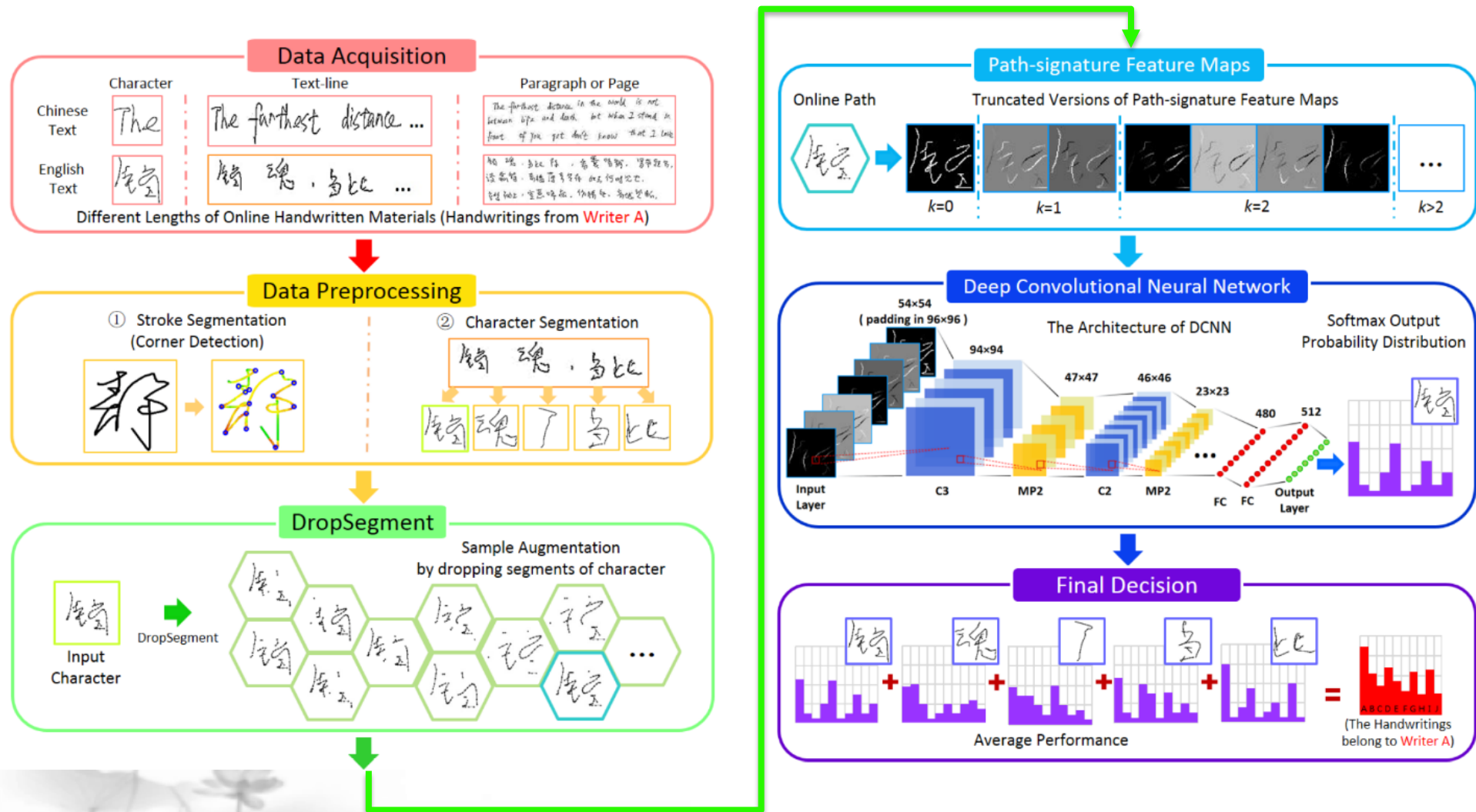
❑ **However, these materials are not always available.**

❑ **We propose two data-augmentation methods, named Drop-Stroke and Drop-Segment, to enrich personal handwriting.**

❑ **We proposed to use CNN to learn efficient discriminant feature based on the bitmap of handwriting text and the **path signature feature** of it**

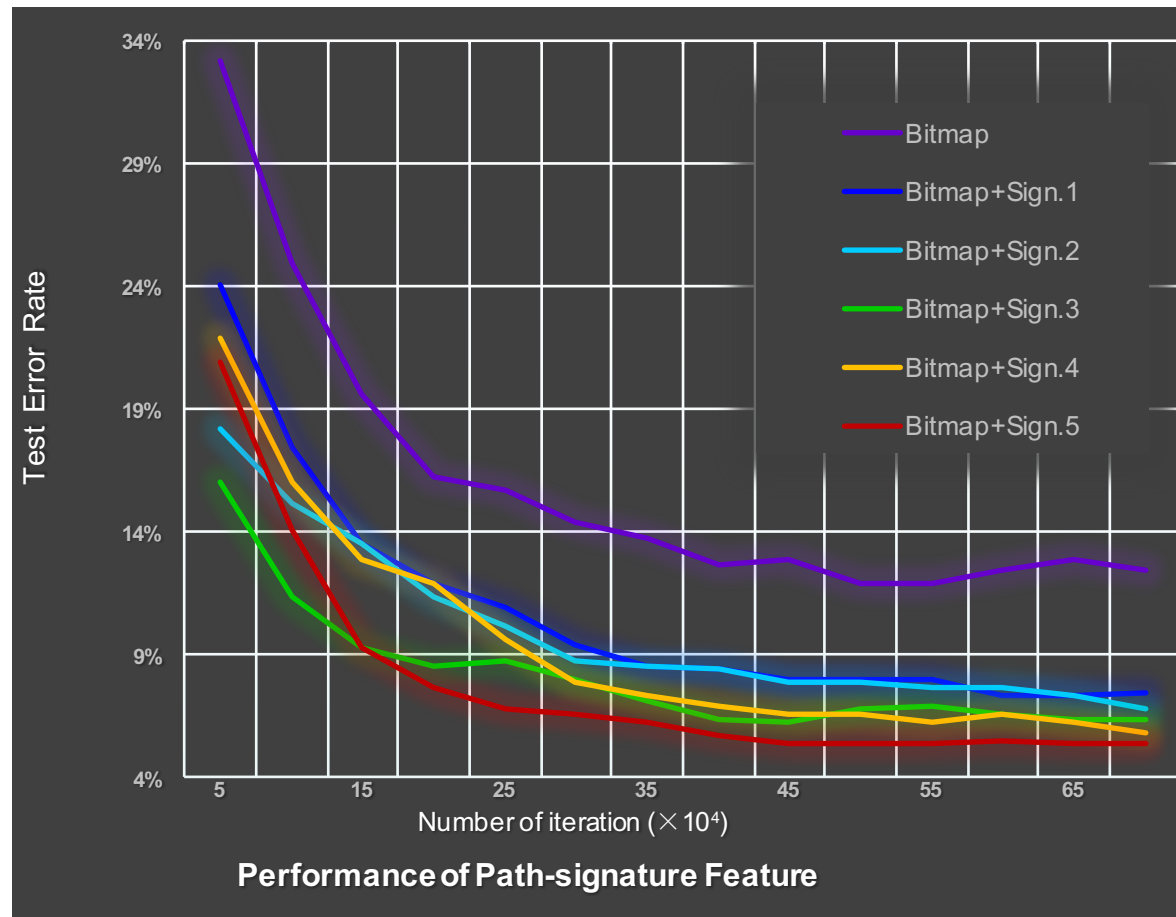


DeepWriterID: system diagram



W.X. Yang, L.W. Jin, et.al., IEEE Intelligent Systems 2016

The Effectiveness of Path Signature Features



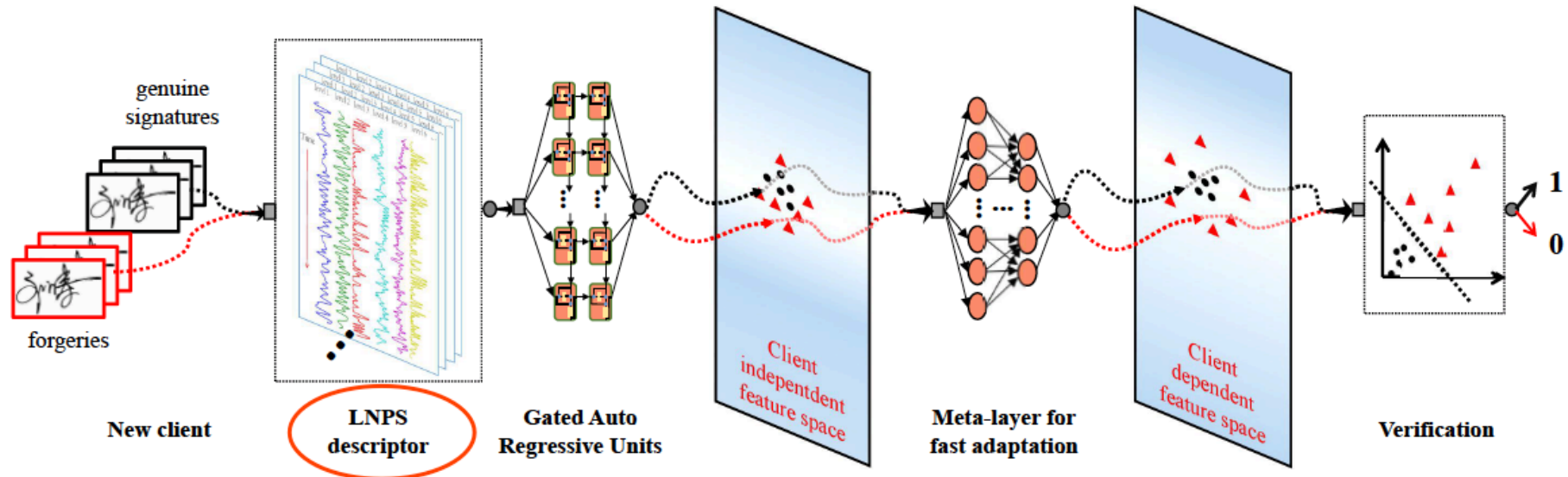
- It can be seen that with the help of signature of path features, the performance is improved significantly

Experimental results

Investigation of Drop-Segment method with different level of Signatures

Methods	Without DropSegment (1 test)	With DropSegment (1 test)	With DropSegment (20 test ensemble)
Baseline: Bitmap (Signature level 0)	87.70	89.30	89.84
Sign. 0+ Sign.1	91.98	93.05	93.05
Sign. (0~2)	92.51	93.05	93.58
Sign. (0~3)	93.05	94.12	94.65
Sign. (0~4)	94.12	94.65	94.12
Sign. (0~5)	94.65	95.19	95.72

Recurrent Adaptation Networks for Online Signature Verification



Main contributions:

- A length-normalized path signature descriptor.
- A meta-layer for fast adaptation.
- A RNN + DTW framework.

Recurrent Adaptation Networks

- Length-normalized path signature (LNPS)
 - Normalize the PS with the path length
 - Leading to scale invariance and a smoothing effect

The k-fold iterated integral:

$$I^k(X) = \int_{0 < t_1 < \dots < t_k < T} dX_{t_1} \otimes \dots \otimes dX_{t_k}.$$

PS, truncated at level m:

$$S(X)|_m = (1, I^1(X), I^2(X), \dots, I^m(X)).$$

LNPS, truncated at level m:

$$S(X)|_m = \left(1, \frac{I^1(X)}{L(X)}, \frac{I^2(X)}{L^2(X)}, \dots, \frac{I^m(X)}{L^m(X)}\right).$$



Recurrent Adaptation Networks

- Effectiveness of Length-normalized path signature (LNPS)

Table 1. Verification results using DTW on the MCYT-100^[1] dataset.

Truncated level	Feature	SF-EER (%)
1	PS	4.43
	LNPS	3.13
2	PS	3.78
	LNPS	2.35
3	PS	4.00
	LNPS	1.86

[1] Ortega-Garcia J, Fierrez-Aguilar J, Simon D, et al. MCYT baseline corpus: a bimodal biometric database[J]. IEE Proc.-Vision, Image and Signal Processing, 2003, 150(6): 395-401.

Length-normalized Path Signature

Szabo Szabo Szabo Szabo Szabo Szabo Szabo

Szabo Szabo Szabo Szabo Szabo Szabo Szabo

Szabo Szabo Szabo Szabo Szabo Szabo Szabo

Juhász Juhász Juhász Juhász Juhász Juhász Juhász

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1-st level

2-nd level



Recurrent Adaptation Networks for Online Signature Verification

Database	Method	SF-EER (%)	Error Reduction
MCYT-100 [1]	Previous SOTA [4]	2.15	15.8%
	Our method	1.81	
e-Biosign [2]	Previous SOTA [2]	11.9	41.6%
	Our method	6.94	
Mobisig [3]	Previous SOTA [3]	25.45	57.3%
	Our method	10.87	

[1] Ortega-Garcia J, et al. MCYT baseline corpus: a bimodal biometric database[J]. 2013

[2] Tolosana R, et al. Benchmarking desktop and mobile handwriting across COTS devices: The e-BioSign biometric database[J]. 2017

[3] Antal M, et al. Online Signature Verification on MOBISIG Finger-Drawn Signature Corpus[J]. 2018.

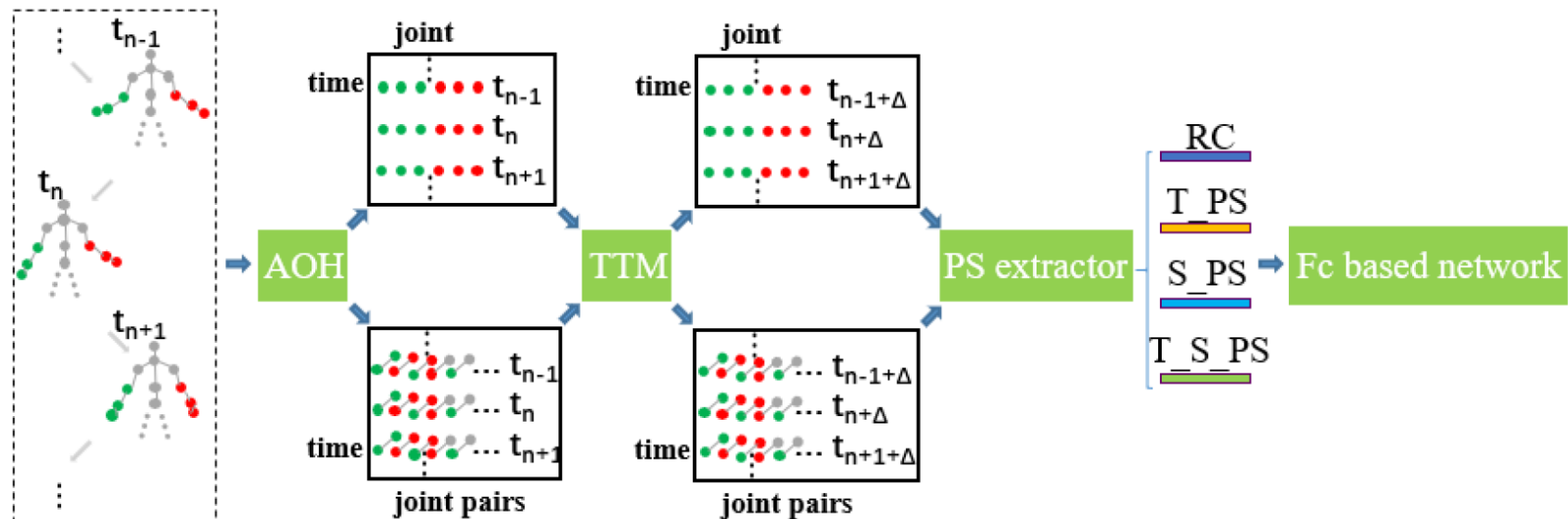
[4] Xia X , et al. Signature alignment based on GMM for on-line signature verification[J]. PR, 2017.

Path Signature for Gesture Recognition



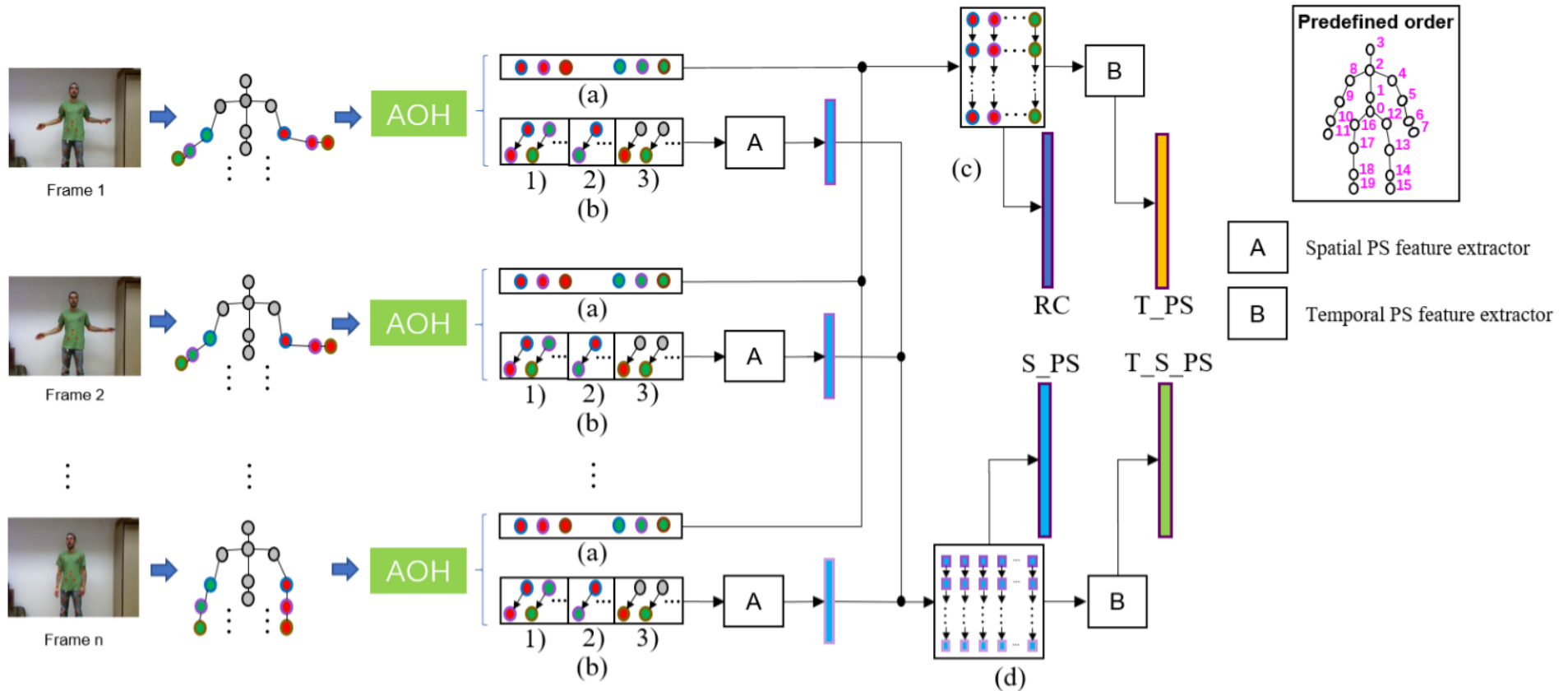
Skeleton-based hand gesture recognition

- Our goal is to recognize skeleton joints trajectories
- Approaches
 - AOH: Attention on Hand
 - TTM: Temporal Transformer Module
 - PS: Spatial & Temporal Path Signatures



C. Li, X Zhang, L. Liao, L. Jin, W Yang, "Skeleton-based Gesture Recognition Using Several Fully Connected Layers with Path Signature Features and Temporal Transformer Module", **AAAI 2019**

Attention on hand (AOH) & Path Signature (PS) Extraction



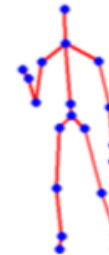
- AOH:**

- **Single joint (a)**
- **Joint pairs (b):**
 - Same hand part
 - Cross hand parts,
 - Cross hand part and upper body part

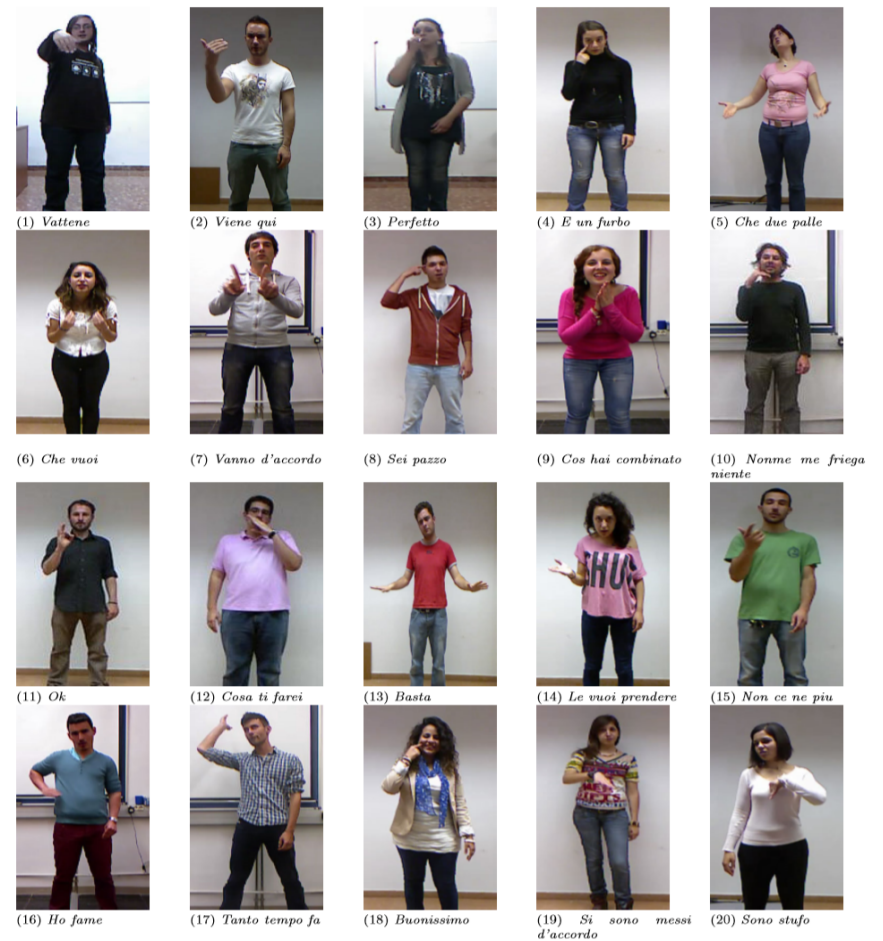
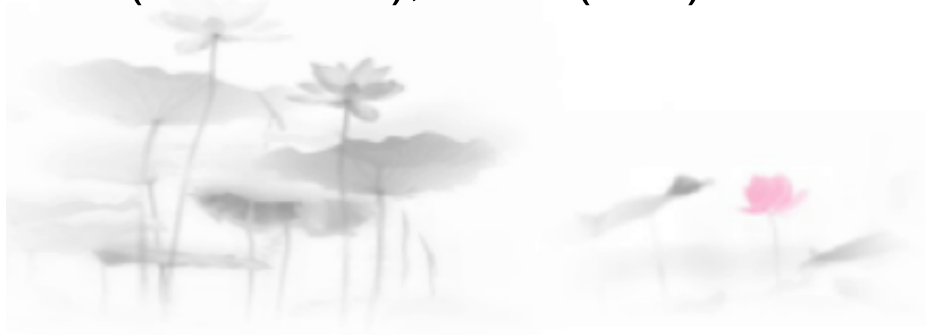
- PS feature:**

- **Spatial PS (S_PS)**
- **Temporal PS (T_PS)**
- **Temporal spatial PS (T_S_PS)**

Experiments



- **Gesture types:** 20 Italian gestures
- **Modality:** RGB, Depth, Mask, Skeleton
- **Samples:** 6850 (train), 3454 (validation), 3579 (test)



Experiments

- Comparison with the state-of-the-arts on ChaLearn 2013, ChaLearn 2016 and MSRC-12

Method	Accuracy rate (%)
HiVideoDarwin Wang, Wang, and Wang	74.90
VideoDarwin Fernando et al.	75.30
D-LSDA Su et al.	76.80
CNN for Skeleton Du, Fu, and Wang	91.20
Two-stream LSTM Wang and Wang	91.70
3s_net_TTM	92.08

Table 5: Comparison of methods on the ChaLearn 2013.

Method	Accuracy rate (%)
JTM Wang et al.	93.12
ESM Jung and Hong	96.76
MP Garcia-Hernando and Kim	98.25
3s_net_TTM	99.01

Table 8: Comparison of methods on the MSRC-12 dataset.

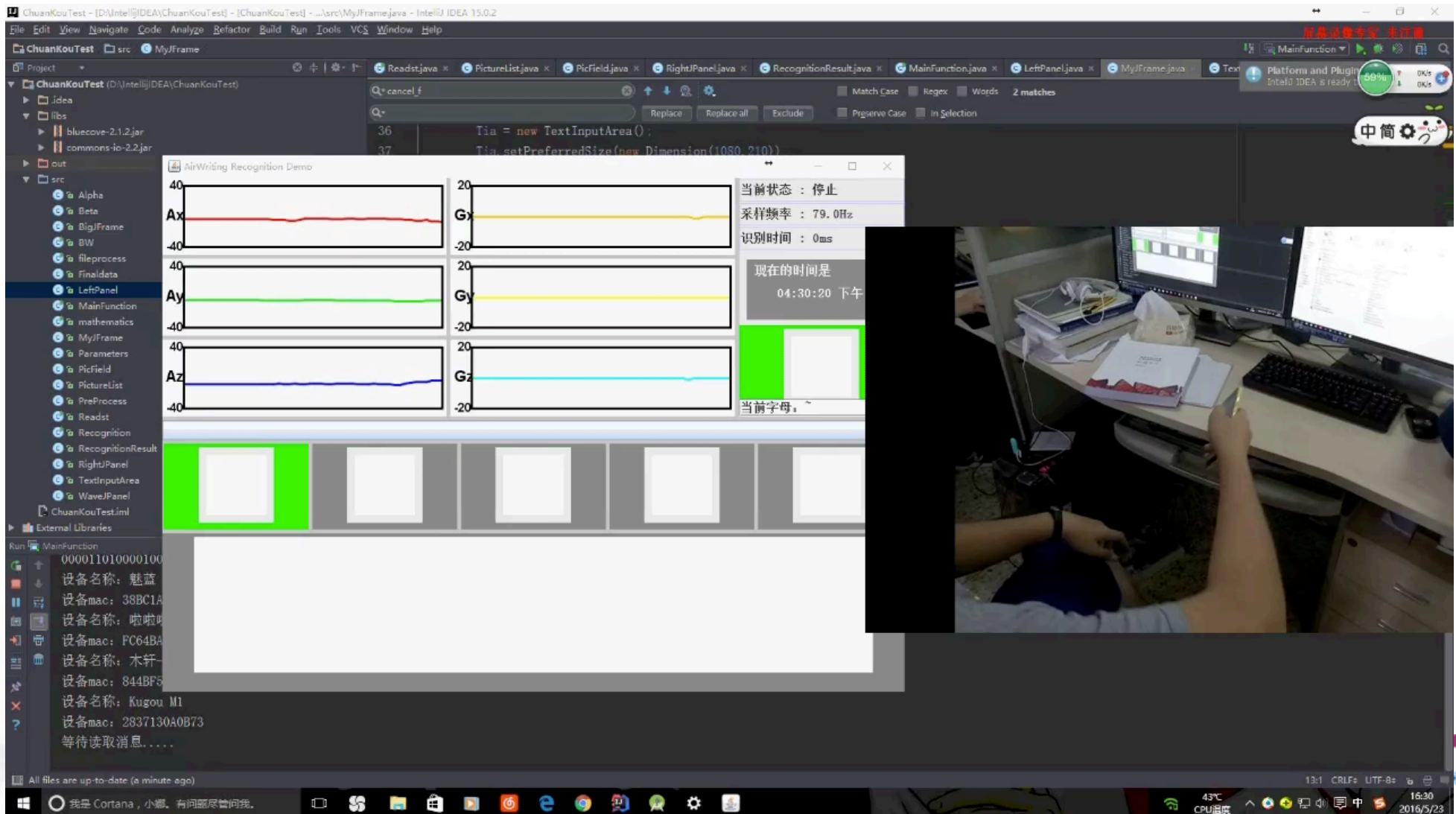
Method	Test acc. (%)	Modality				Model
		R	D	O	S	
AMRL ^[1]	65.59	✓	✓			8*CNN+4ConvLSTM
ASU ^[2]	67.71	✓	✓	✓		4*C3D+2*TSN+1*SVM
FOANet ^[3]	82.07	✓	✓	✓		12*CNN
SkeLSTM ^[4]	35.39				✓	1*LSTM
3s_net_TTM	39.95				✓	3*FC

Table 7: Comparison on ChaLearn 2016 dataset. ^[1]: Wang et al. ^[2]: Miao et al. ^[3]: Narayana, Beveridge, and Draper ^[4]: Lin et al. RGB, depth, optical flow and skeleton are abbreviated as R, D, O and S.



More Application of PS...

- Path Signature for activity recognition



Demo



Video Demo



Thanks

Thanks!

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